Spread Pilots

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Abstract: will describe how to use filters in the discrete delay-Doppler domain to create noise-like waveforms with excellent PAPR (around 5dB).

Background - Zak-OTFS and Integration of Sensing and Communication– in collaboration with Muhammad Ubadah, Saif Khan Mohammed, Ronny Hadani, Shachar Kons, and Ananthanarayanan Chockalingam

Disclosure: Advisor to Cohere Technologies

Symplectic Transforms in Radar

Displacement operator: $D(\tau, \nu)\ \phi(t) = e^{-\nu\tau} e^{j\nu t} \phi(t+\tau)$

Self-ambiguity function: $A_\phi(\tau, \nu) = \langle \phi\ ,\ D(\tau, \nu)\ ,\ \phi \rangle$

Symplectic transform: For example, Linear Frequency Modulation (LFM)

Chirp with slope q: $U(q)\ :\ \ \phi(t) \rightarrow \phi_q(t) = e^{j\,q\,t^2}\,\phi(t)$

Symplectic transforms rotate self-ambiguity functions

$$A_{\phi_q}(\tau, \nu) = \langle \phi_q, D(\tau, \nu)\,, \phi_q \rangle = \langle \phi\,, U(q)^{+}\, D(\tau, \nu)\, U(q)\,, \phi \rangle$$

$$A_{\phi_q}(\tau, \nu) = \left\langle \phi\,, D\left[S_q^{-1}\begin{pmatrix}\tau \\ \nu\end{pmatrix}\right], \phi \right\rangle \ with\ S_q = \begin{bmatrix} 1 & 0 \\ 2q & 1 \end{bmatrix}$$

S. D. Howard et. al. (2004), Waveform Libraries for Radar Tracking

Discrete DD Domain Filters in Communications

MN-Periodic Discrete Chirp Filter with slope q (coprime to MN)

$$w[k,l] = \frac{1}{MN}\, e^{j\,2\,\pi\frac{q\left(k^2+l^2\right)}{MN}}, \quad for\ \ k,l \in \mathbb{Z}$$

Spread pilot: $x_{s,dd}[k,l] = w_s[k,l] \ *_\sigma\ x_{p,dd}[k,l]$

$x_{s,dd}$ is quasi-periodic, $x_{s,dd}$ is periodic with period MN along delay and Doppler axis

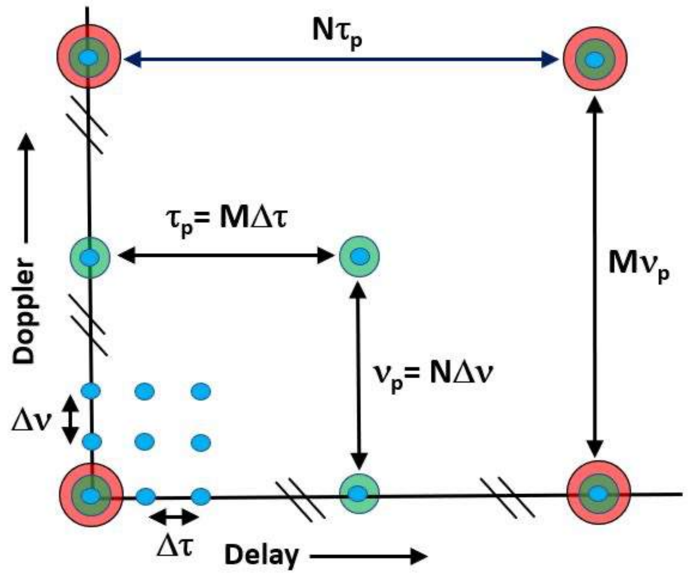
$w[k,l] \ \circledast_\sigma\ x_{p,dd}[k,l]$: MN-Periodic twisted convolution of $x_{p,dd}[k,l]$ with $w[k,l]$ (the MN-periodic extension of the discrete filter)

Theorem: $w_s[k,l] \ *_\sigma\ x_{p,dd}[k,l] = w[k,l] \ \circledast_\sigma\ x_{p,dd}[k,l]$

Degrees of Freedom: a discrete DD domain filter is specified by a vector in $\mathbb{C}^{M^2N^2}$

Chirp Filters Rotate the Period Lattice

Point pulsone x_p : $A_{x_p,x_p}[k,l]$ supported on the period lattice Λ_p



Blue dots: Information lattice Λ_{dd}

Green dots: Period lattice Λ_p

Red dots: Dual lattice $\Lambda_{dd}^{\perp}$

Spread pulsone x_s

Theorem: Ambiguity function $A_{x_s,x_s}[k,l]$ is supported on a rotated lattice Λ_q given by

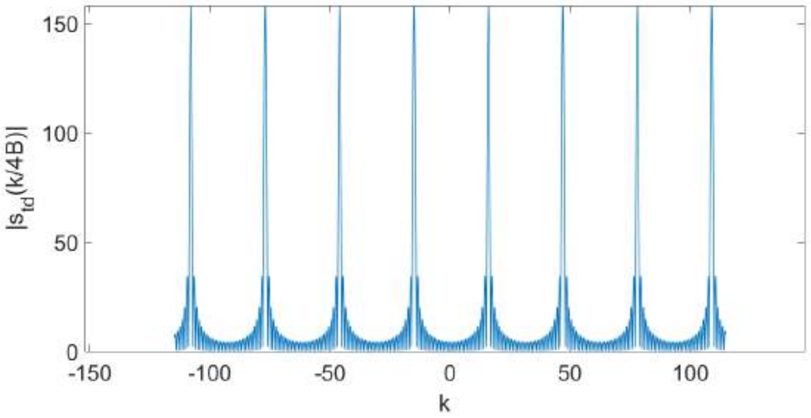
$$(1 - 2q\,\theta)\ \Lambda_q = \begin{bmatrix} \theta & 1 \\ 1 & 2q \end{bmatrix} \Lambda_p \quad \text{where } \theta = (2q)^{-1} - 2q \pmod{MN}$$

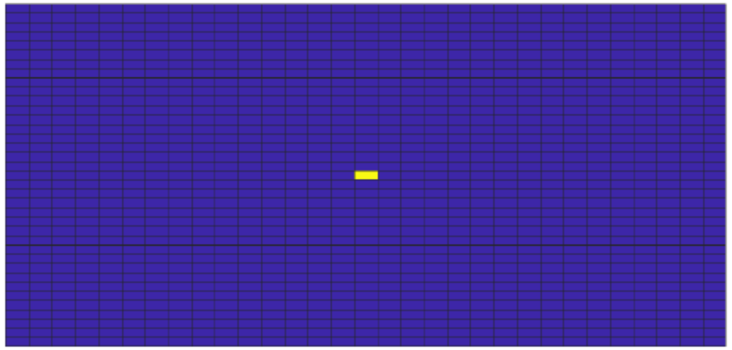
Spreading in TD and DD domain

TD

DD domain

x_p



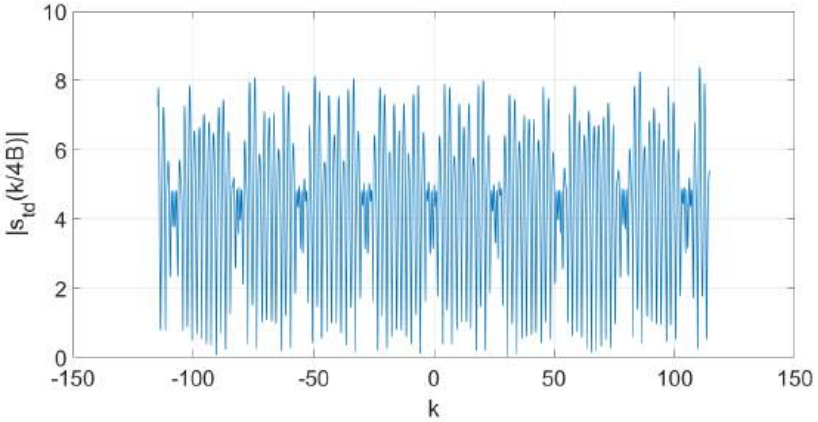


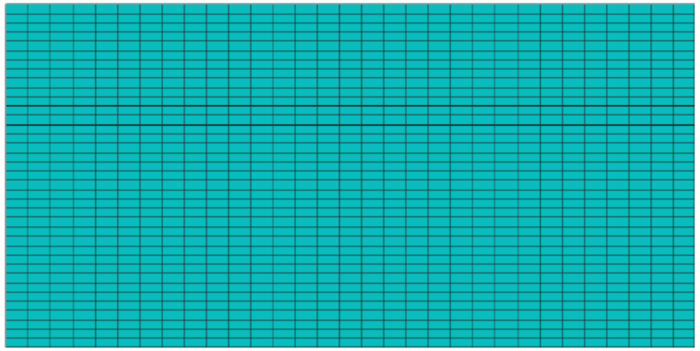
peaky waveform

energy focussed at pilot location

Chirp filter with slope $q = 3$

x_s

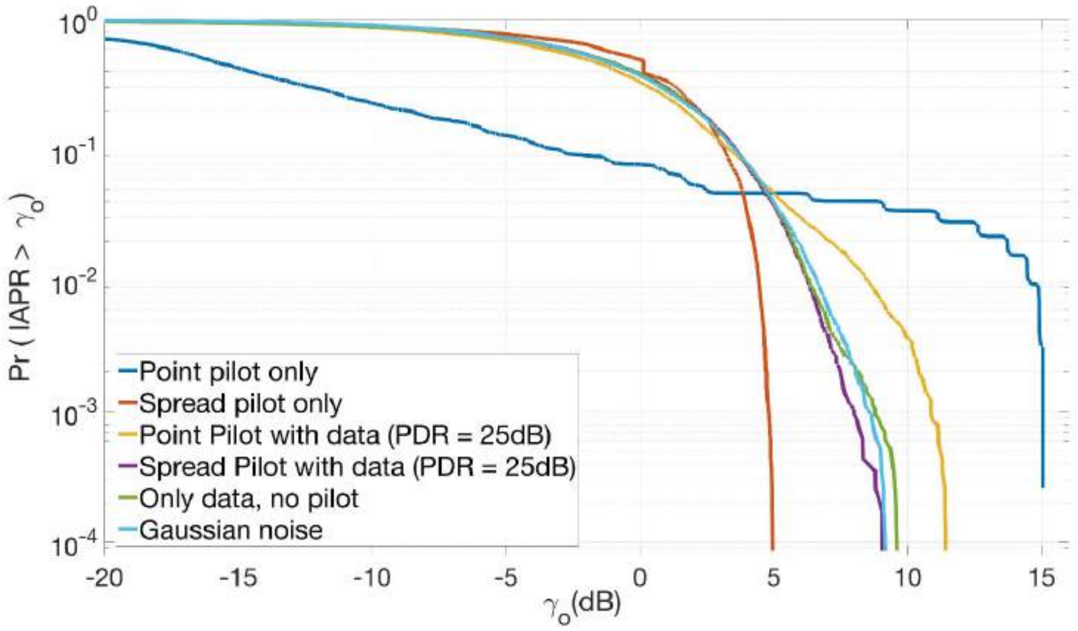




noise-like waveform

Energy $\left|x_{s,dd}[k,l]\right|^2 \approx \frac{1}{MN}$

Peak to Average Power Ratio (PAPR)



Complementary CDF (CCDF) of Instantaneous to Average Power Ratio (IAPR) : RRC transmit filter with roll-off factors $\beta_\tau = \beta_\nu = 0.6.$

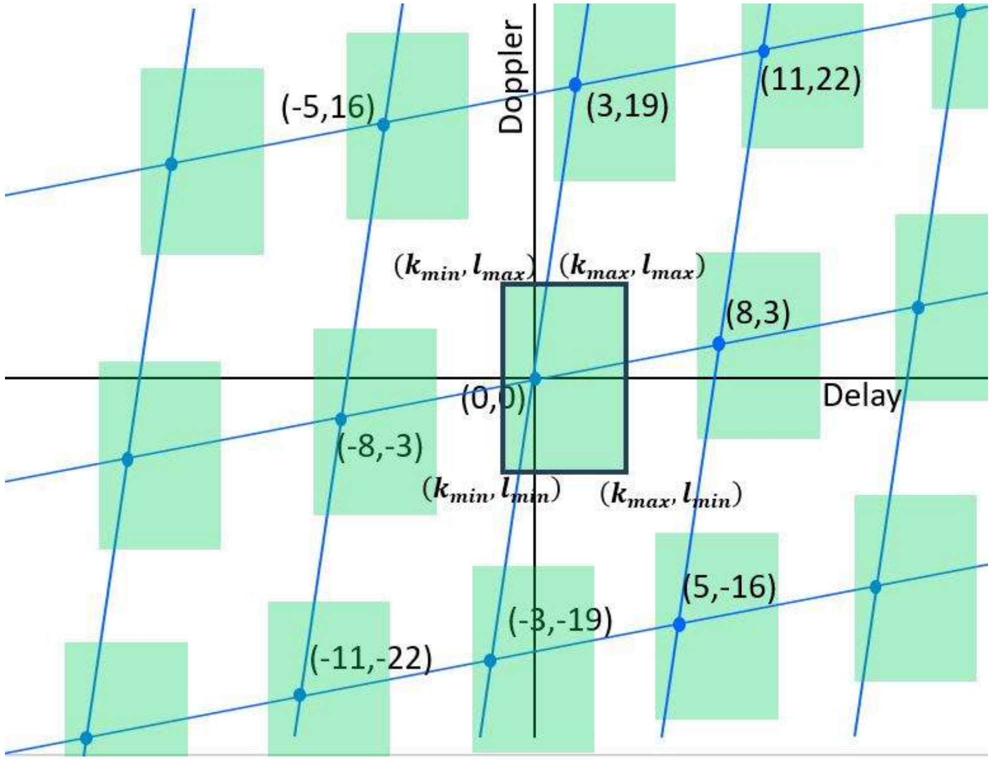
IAPR Summary :

Spread pulsone $\sim$ 5 dB vs. point pulsone $\sim$ 15 dB

Data + spread pulsone $\sim$ 8 dB vs. Data + point pulsone $\sim$ 12 dB

Geometry of Sensing

Blue dots: Λ_q



$M = 11, N = 13, q = 5$

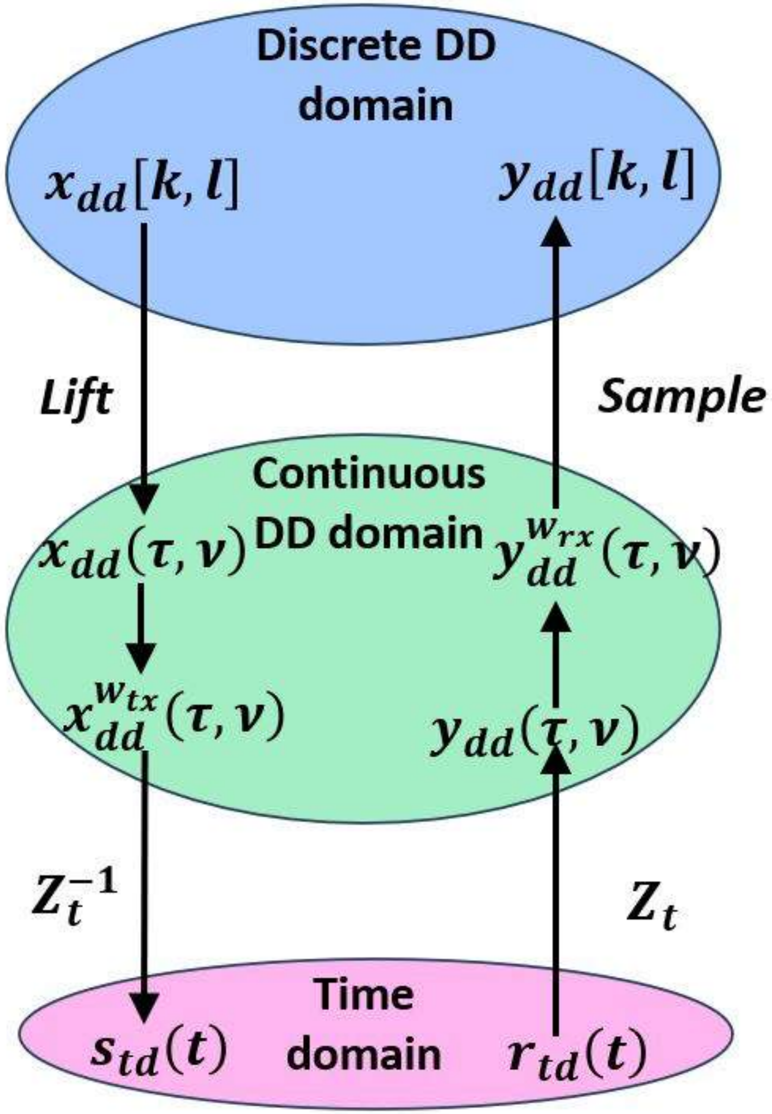
Pilot location: $(k_p\ ,\ l_p) = (0\ ,0\ )$

Green rectangle with black border (Ω): support of $h_{\mathrm{eff}}[k, l]$

Translates of Ω by Λ_q do not overlap : No DD domain aliasing

Possible to accurately estimate $h_{\mathrm{eff}}[k, l]$ on Λ_{dd} from the response received within Ω

Filters in the Discrete DD Domain



Summary

Possible to construct spread waveforms with desirable characteristics by applying a chirp filter in the discrete DD domain to a point pulsone

Low PAPR: about 5 dB versus 15 dB for the point Pulsone

Possible to read off the I/O relation provided a second crystallization condition is satisfied w.r.t. Λ_q