Zak-OTFS and LDPC Codes

Robert Calderbank

Duke University

Abstract: It is possible to read off the effective DD domain channel filter from the response to a single Zak –OTFS pilot. The more reliable estimates concentrate around the pilot, and we use this to optimize LDPC code design.

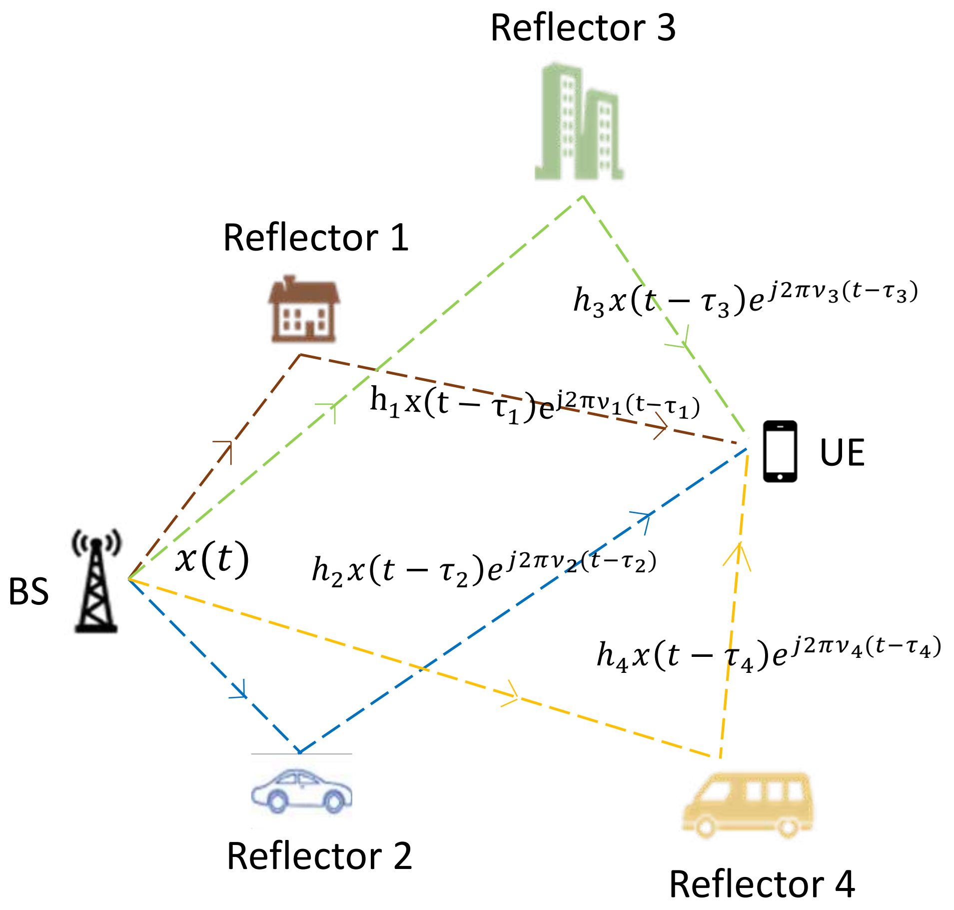
This is joint work with Beyza Dabak and Venkatesh Khammammetti

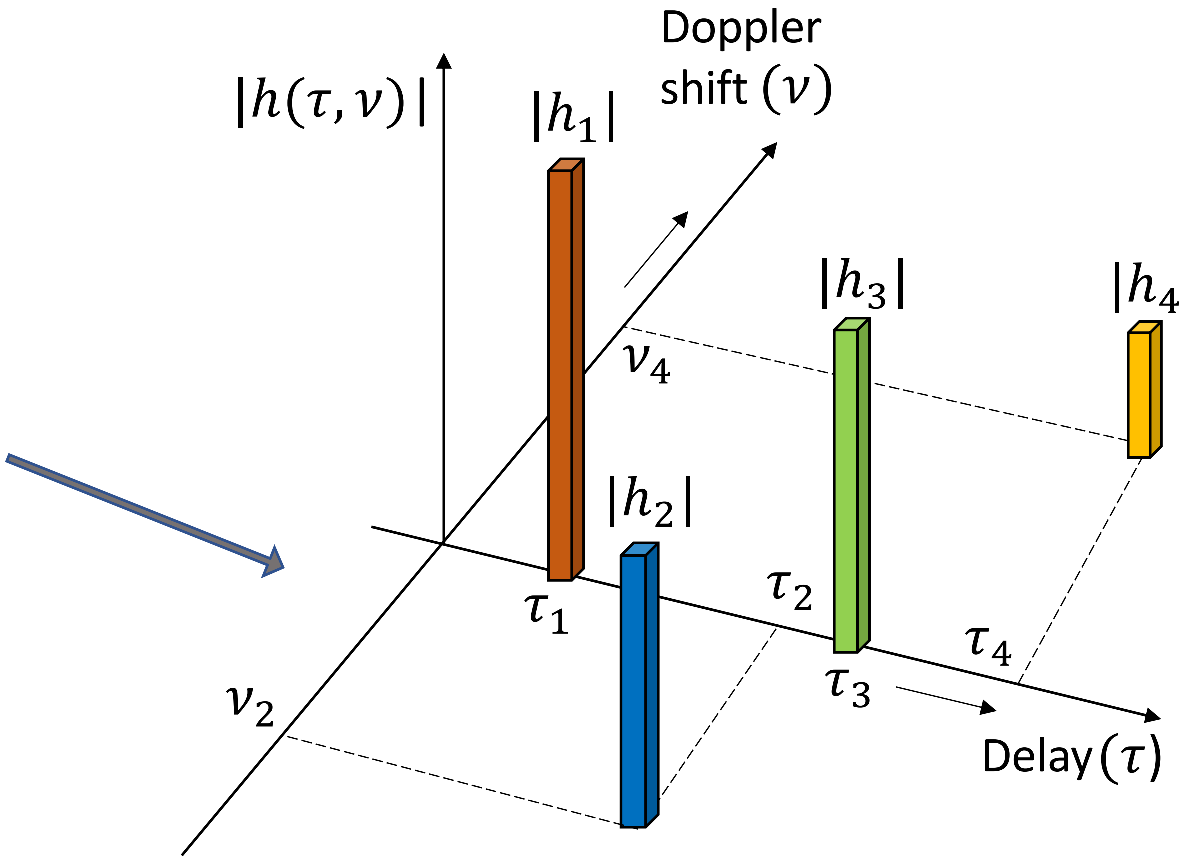
Learn More - IEEE BITS Magazine: *A Mathematical Foundation for Communications and Sensing in the Delay-Doppler Domain, Parts I and II* – in collaboration with Saif Khan Mohammed, Ronny Hadani, and Ananthanarayanan Chockalingam

Disclosure: Advisor to Cohere Technologies

Representing Doubly Spread Channels

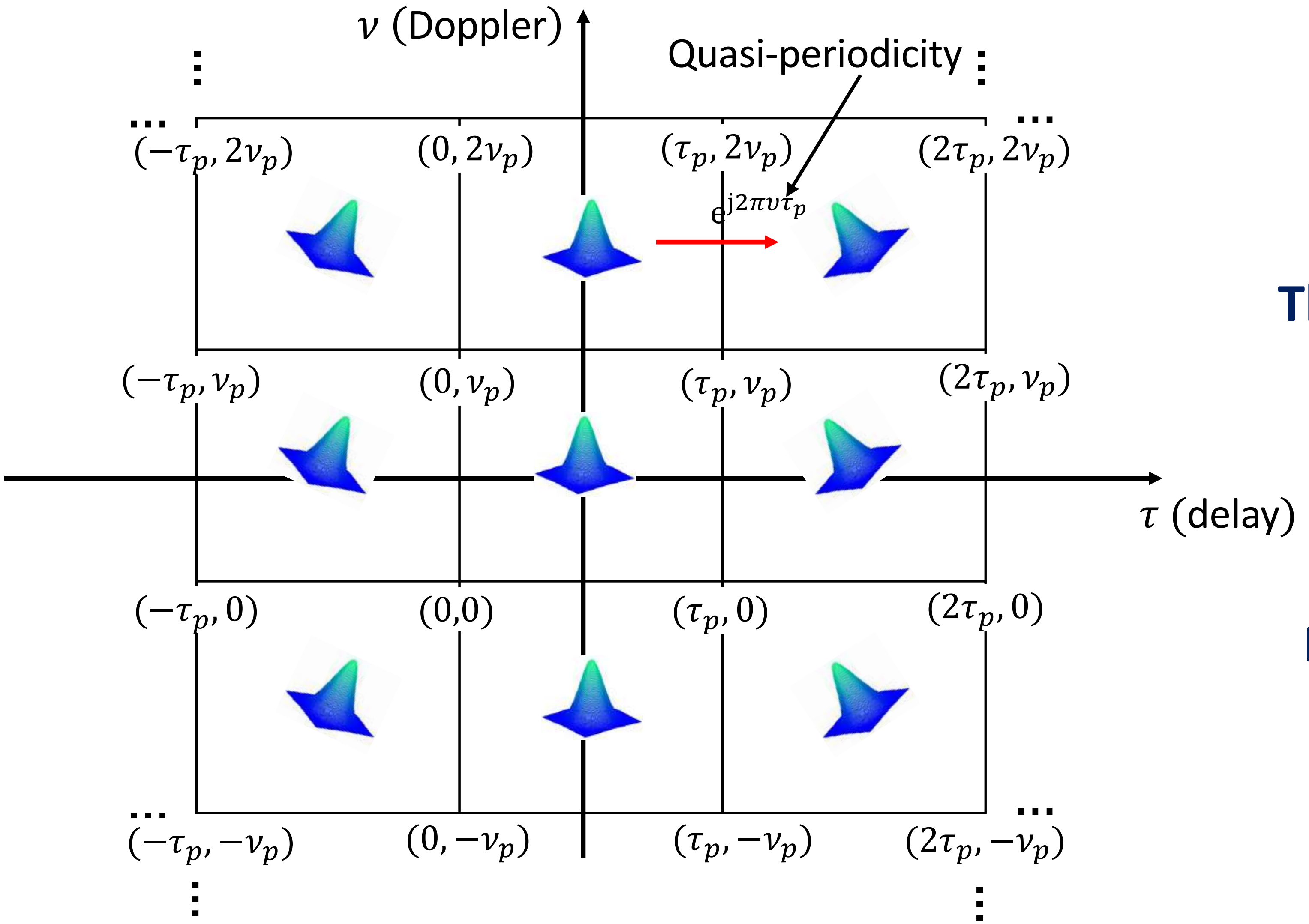
Taps in Delay and Doppler





Delay Spread (T_d) :-	Doppler Spread (D_s) :-
$T_d = \max_{i,j} \lvert \tau_i - \tau_j \rvert$	$D_s = \max_{i,j} \lvert \nu_i - \nu_j \rvert$

A Pulse in the Delay-Doppler Domain

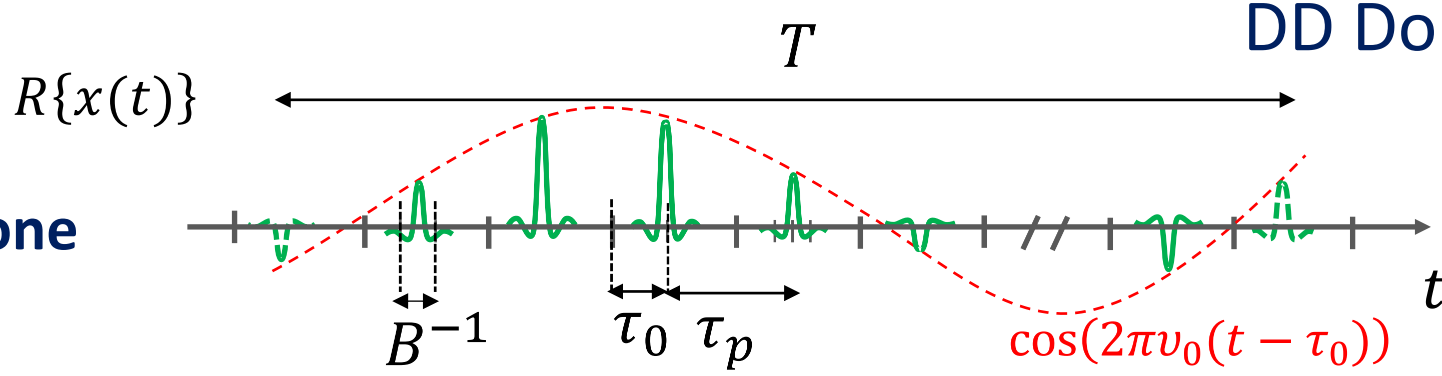


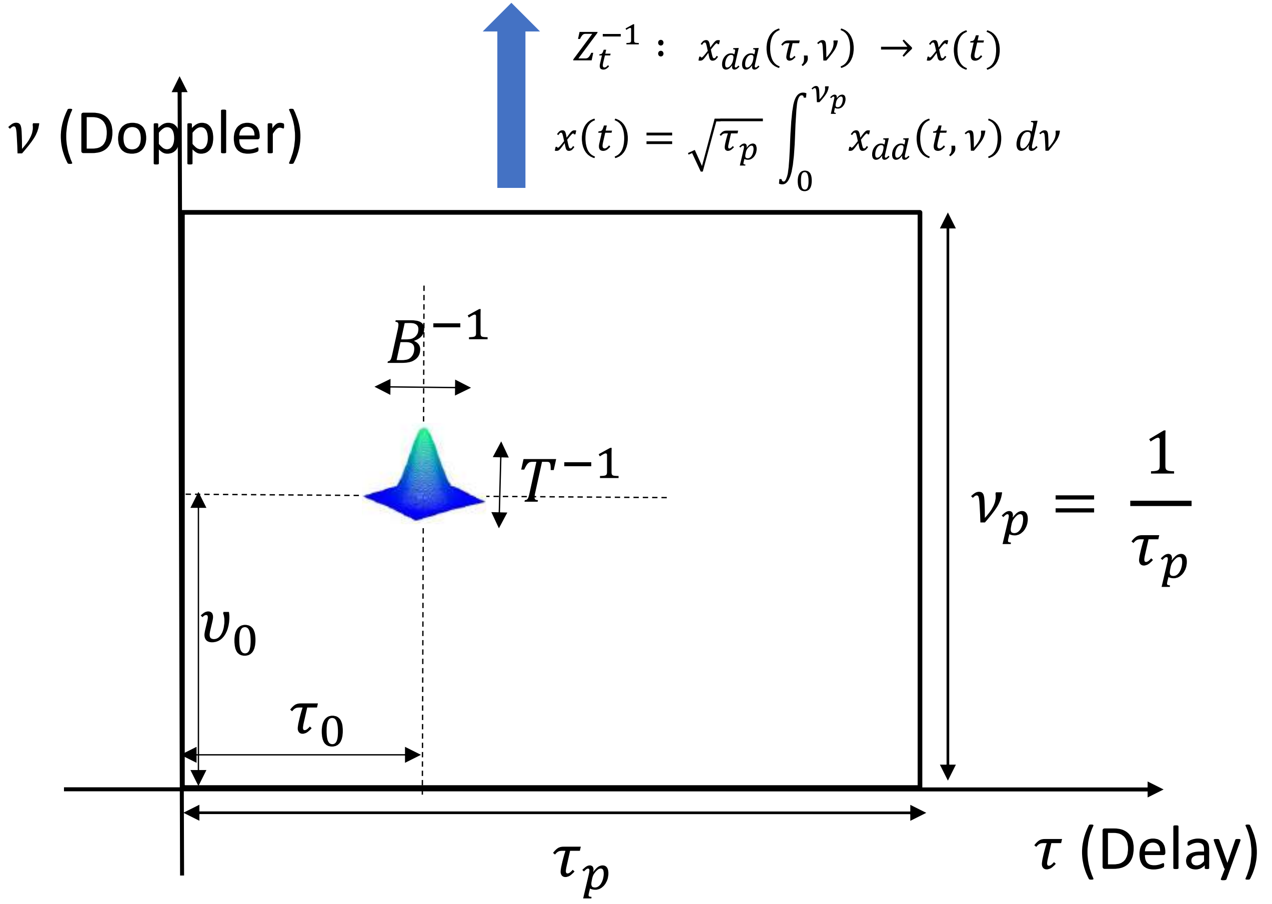
The DD realization of a TD signal is a quasi-periodic function

Fundamental Domain defined by the delay period τ_p and the Doppler period ν_p

TD Pulsone from a Quasi-Periodic DD Domain Pulse

TD Pulsone



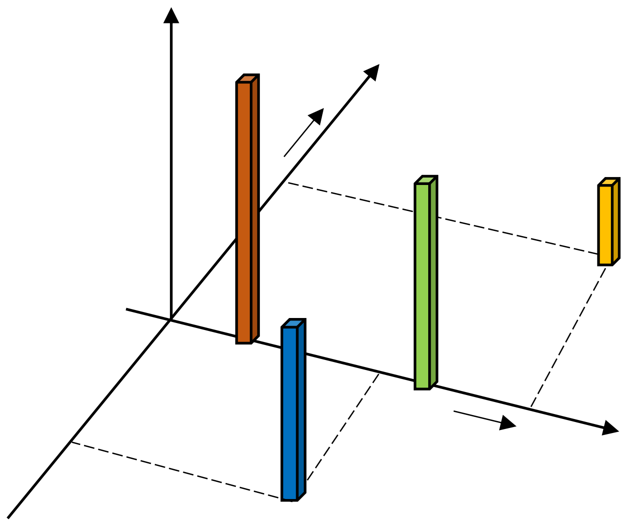


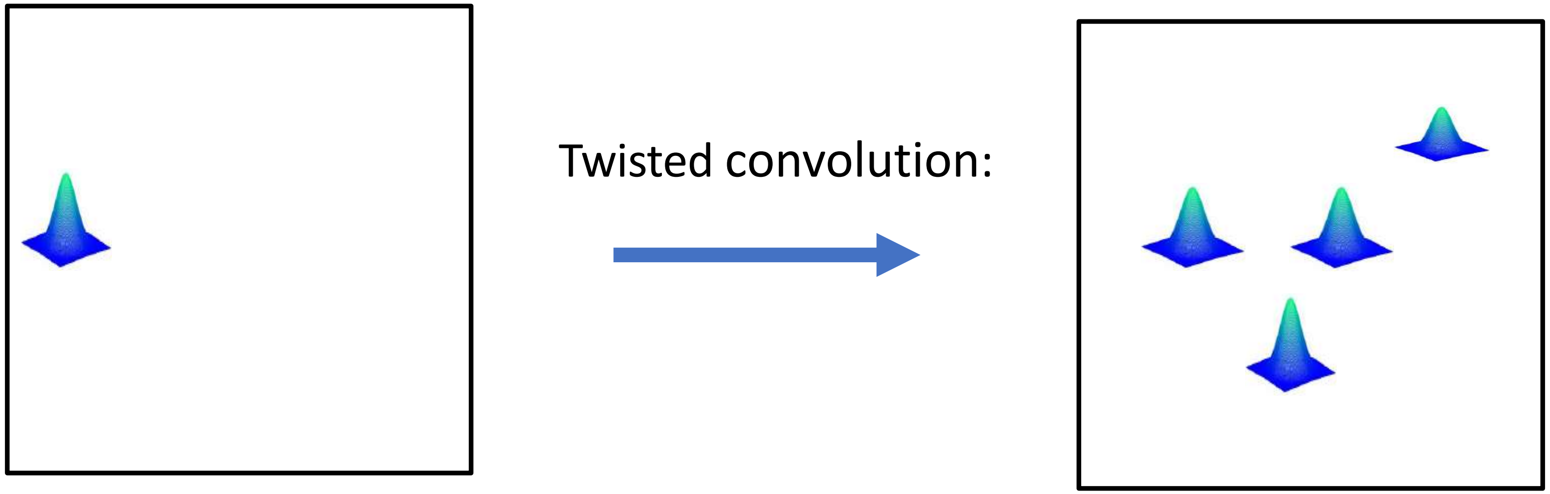
TDM is a limiting case

FDM is a limiting case

Pulsones parametrized by τ_p interpolate between TDM and FDM

Doubly Spread Channels Acting on Pulsones



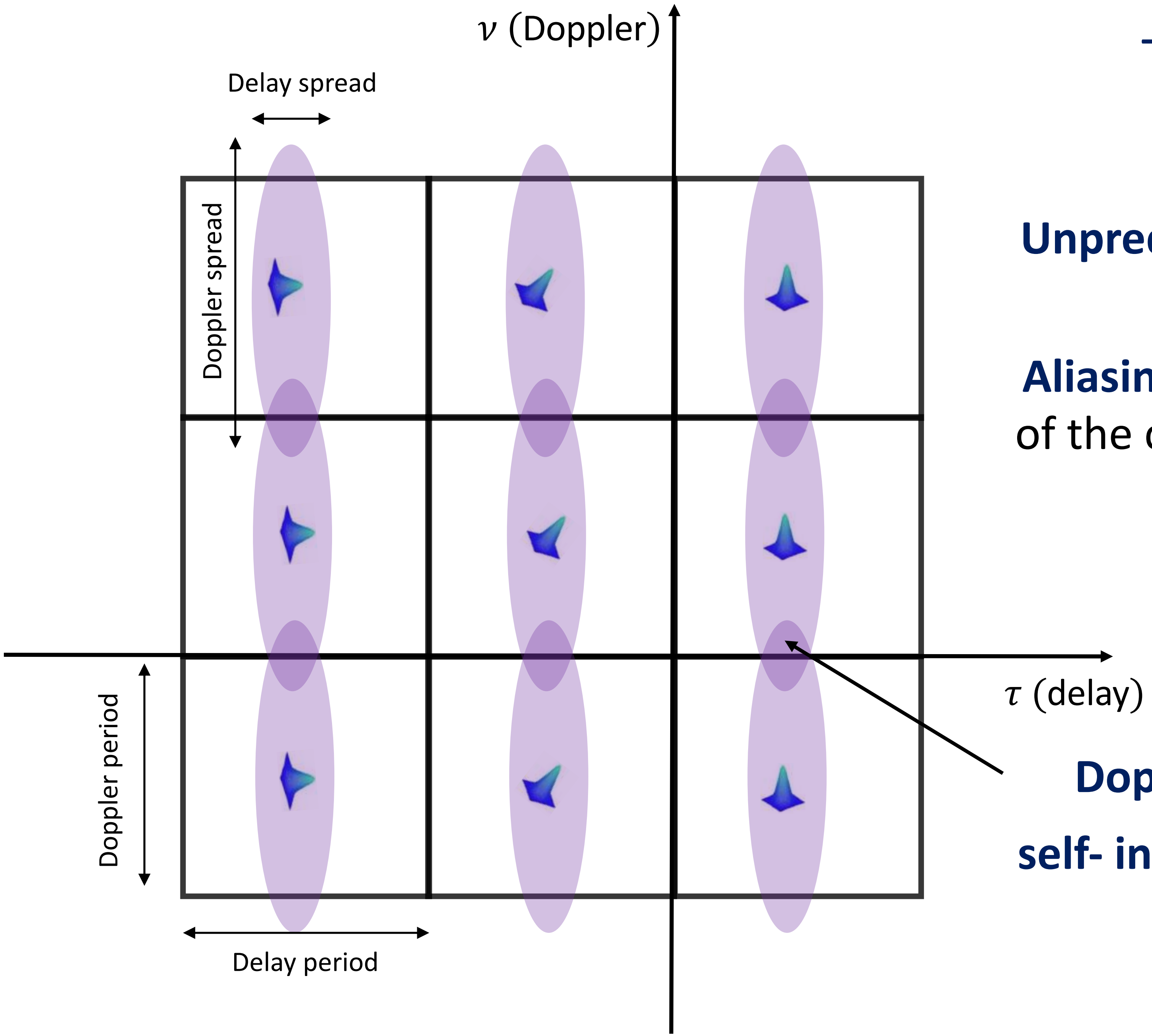


Crystalline Regime: The delay domain period τ_p is greater than the channel path delay spread, and the Doppler domain period ν_p is greater than the path Doppler spread:

$$\tau_p > \text{delay spread} \quad \text{and} \quad \nu_p > \text{Doppler spread}$$

The interaction of a doubly spread channel with a TD pulsone is predictable and geometric

The Origins of Unpredictability



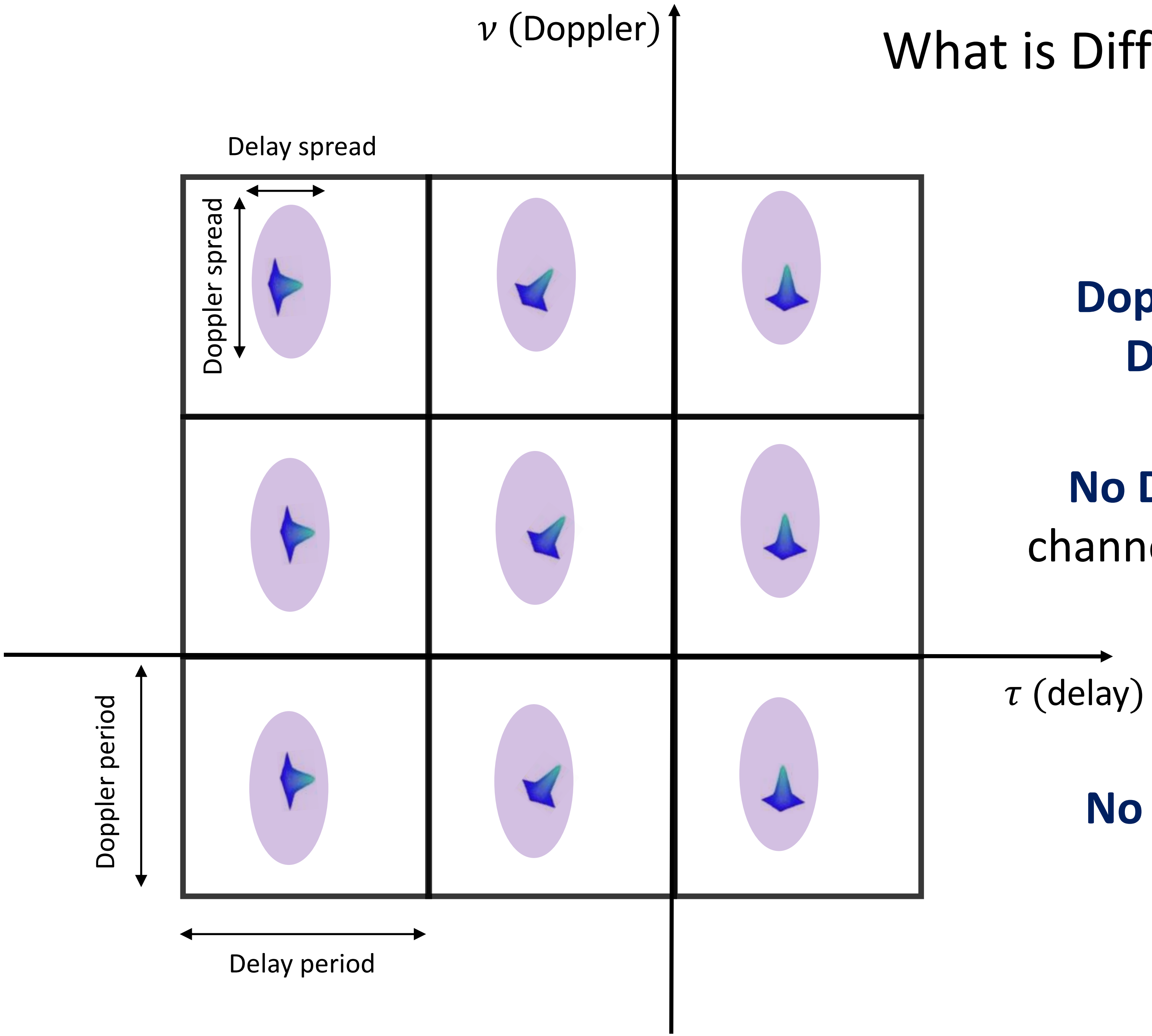
Unpredictability results from aliasing in the DD domain

Aliasing occurs when the DD spreads of the channel are bigger than the DD periods of the pulsone

Doppler spread > Doppler period

self- interaction leads to unpredictability

What is Different in the Crystalline Regime?



Doppler spread < Doppler period
Delay spread < Delay period

No DD Aliasing - the DD spreads of the channel are smaller than the DD periods of the pulsone.

No Self-Interaction means the channel response is predictable

Single Pilot Channel Estimation and RPE

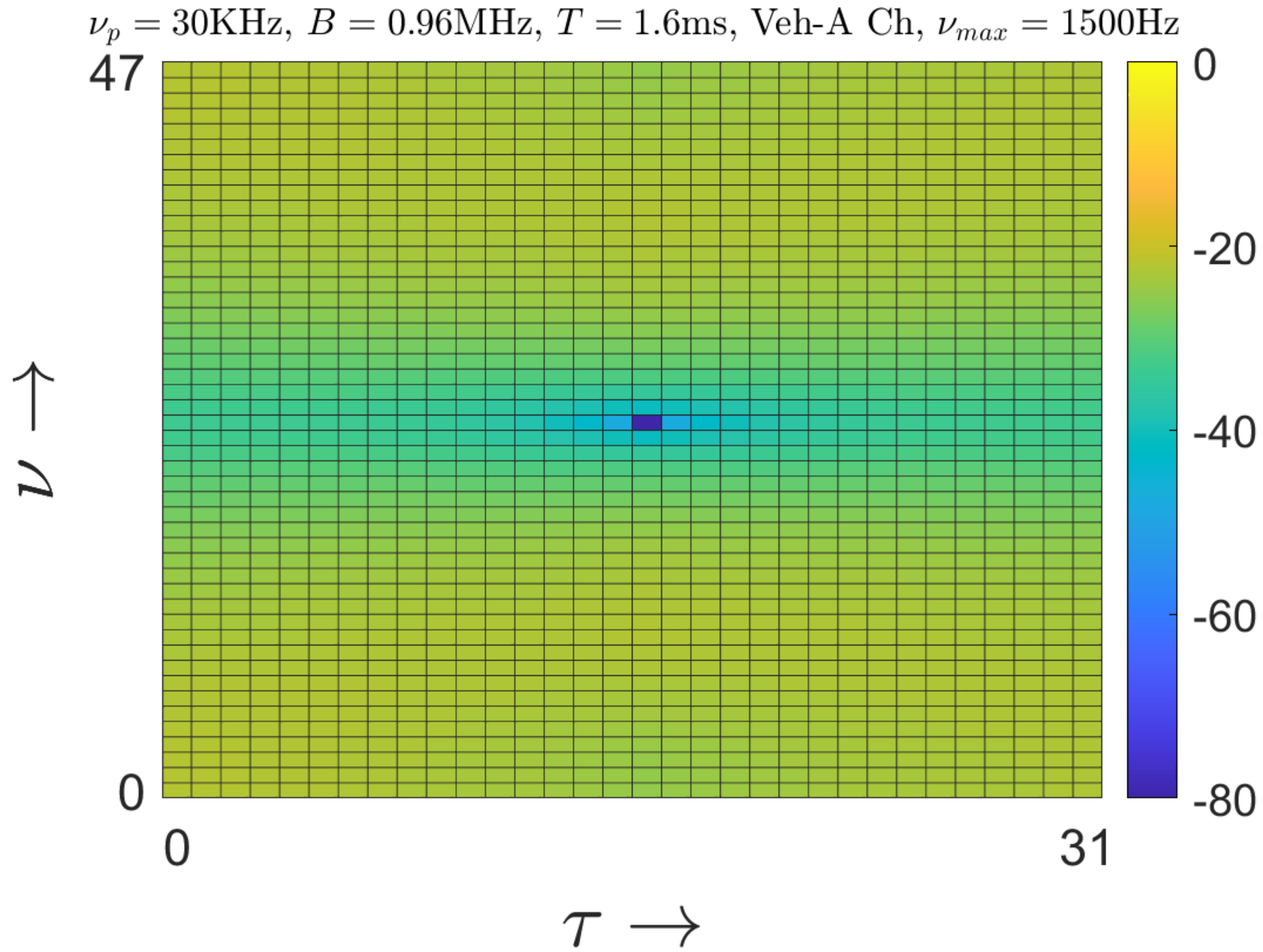
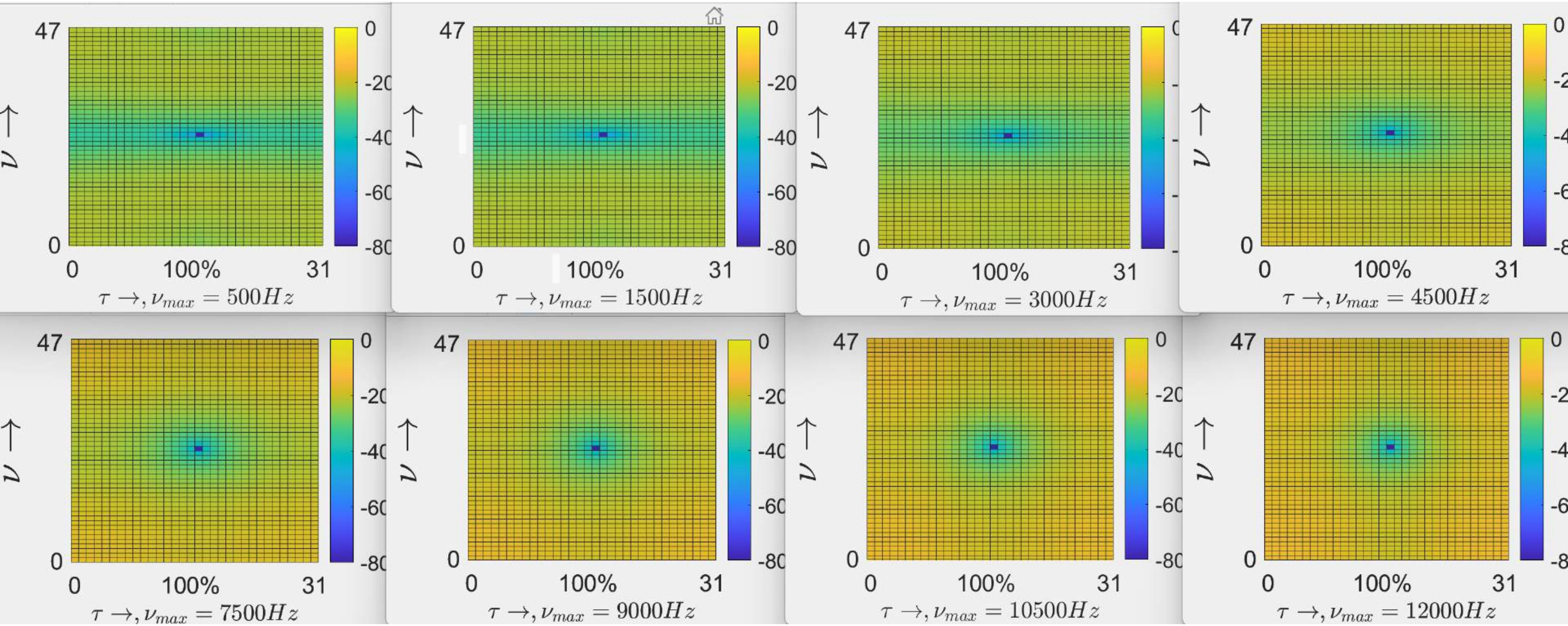


Figure: RPE with Veh-A 6 paths Channel Model

Goal: Exploiting the differences in reliability with coding allocation strategies to extend the range of Doppler spreads for which reliable operation is possible

RPE with Respect to Varying Maximum Doppler Spread



LDPC Coding

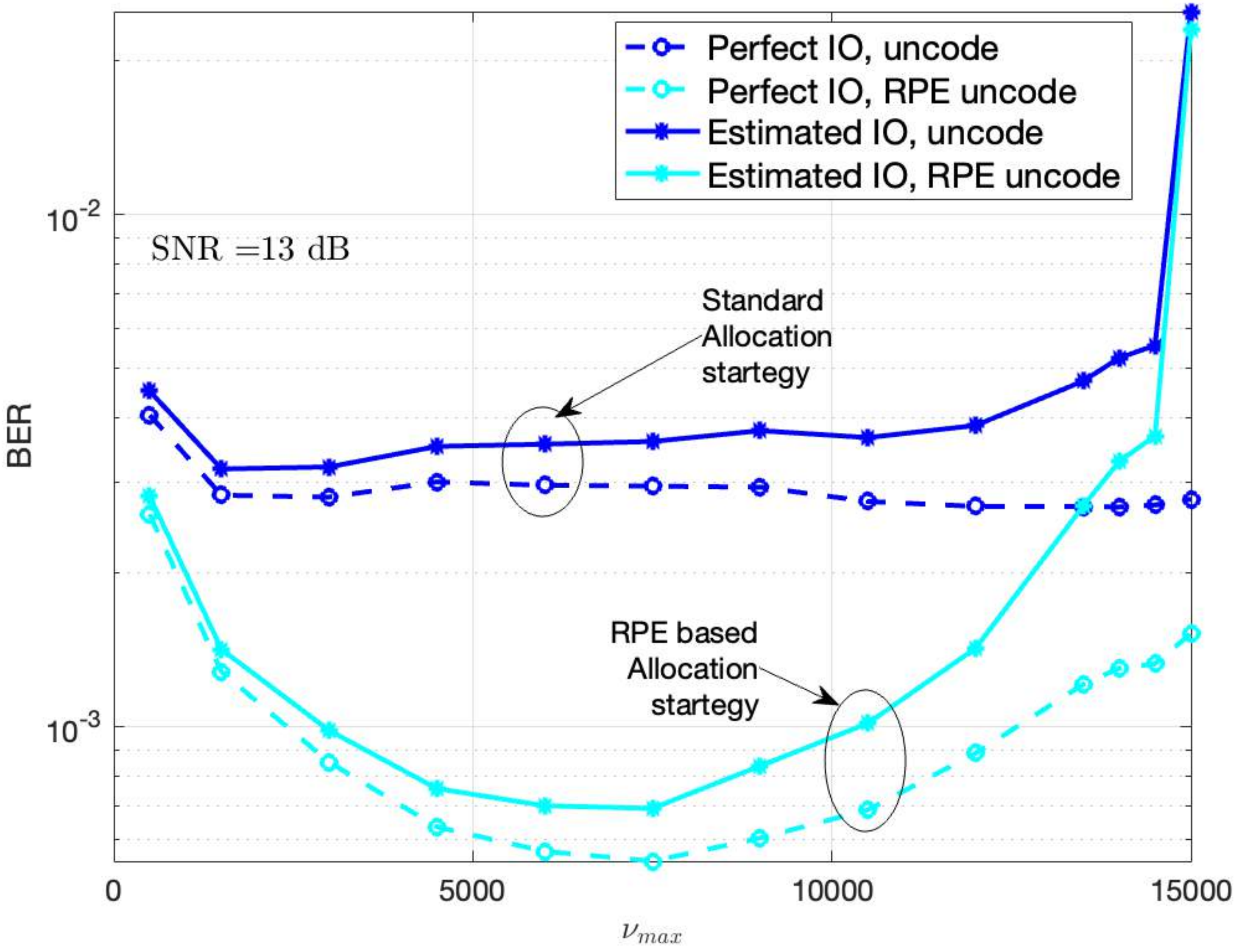
- We send a single pilot at the center of the frame and estimate the channel based on its response at the receiver.
- The received symbols are given by, $\boldsymbol{y} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{n}$
- MMSE equalization on the received symbols, i.e.,

$$\widehat{\boldsymbol{x}} = \left(\boldsymbol{H}^{*}\boldsymbol{H} + diag\left(\sigma_i^2\right)\boldsymbol{I}\right)^{-1}\boldsymbol{H}^{*}\boldsymbol{y}$$

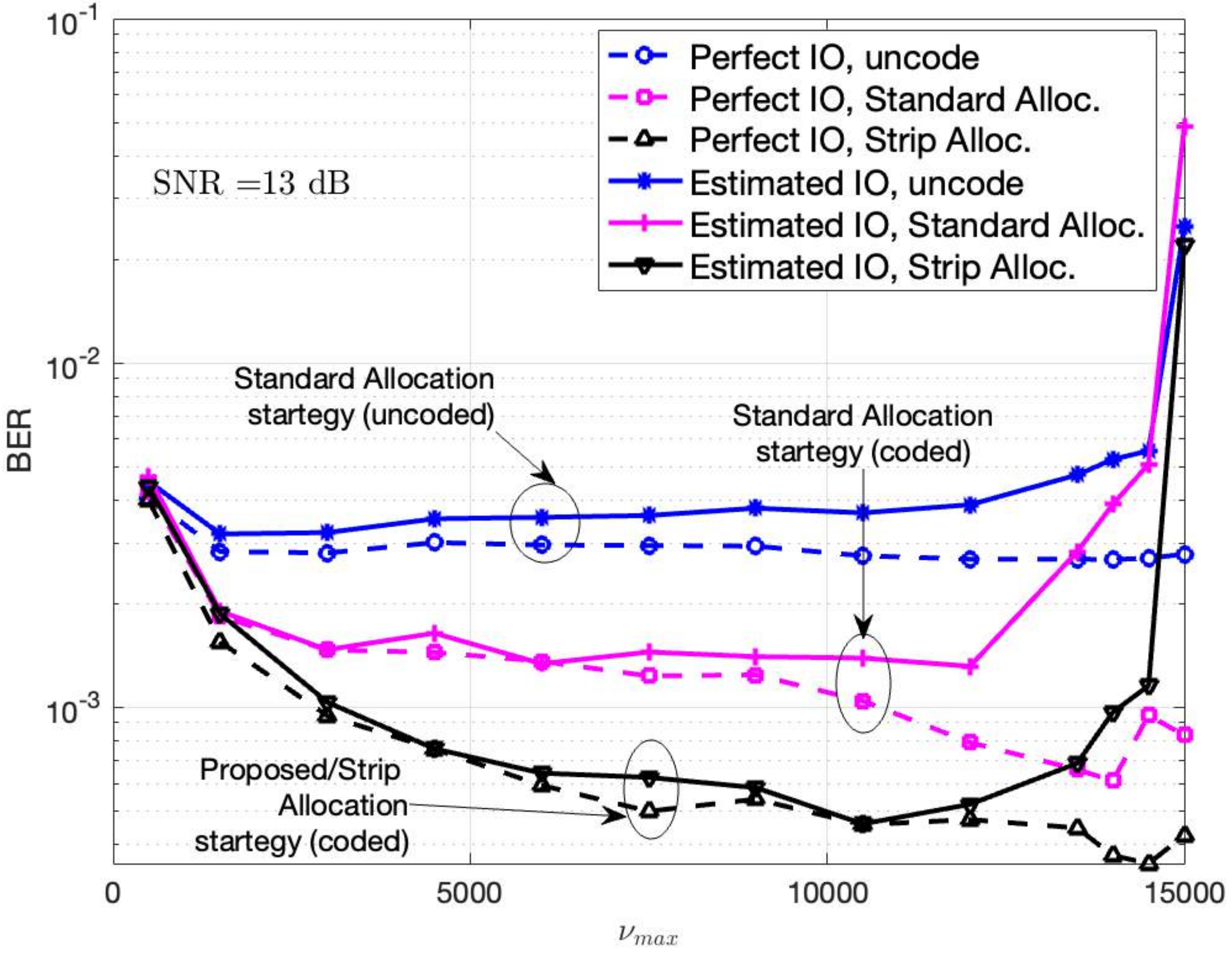
- For uncoded transmission, hard decoding on $\widehat{\boldsymbol{x}}$ and generated received bits
- For coded transmission, LLRs are input to the LDPC decoder

➢ Information bits are mapped to higher reliability regions with respect to RPE, while parity bits are mapped to lower reliability regions

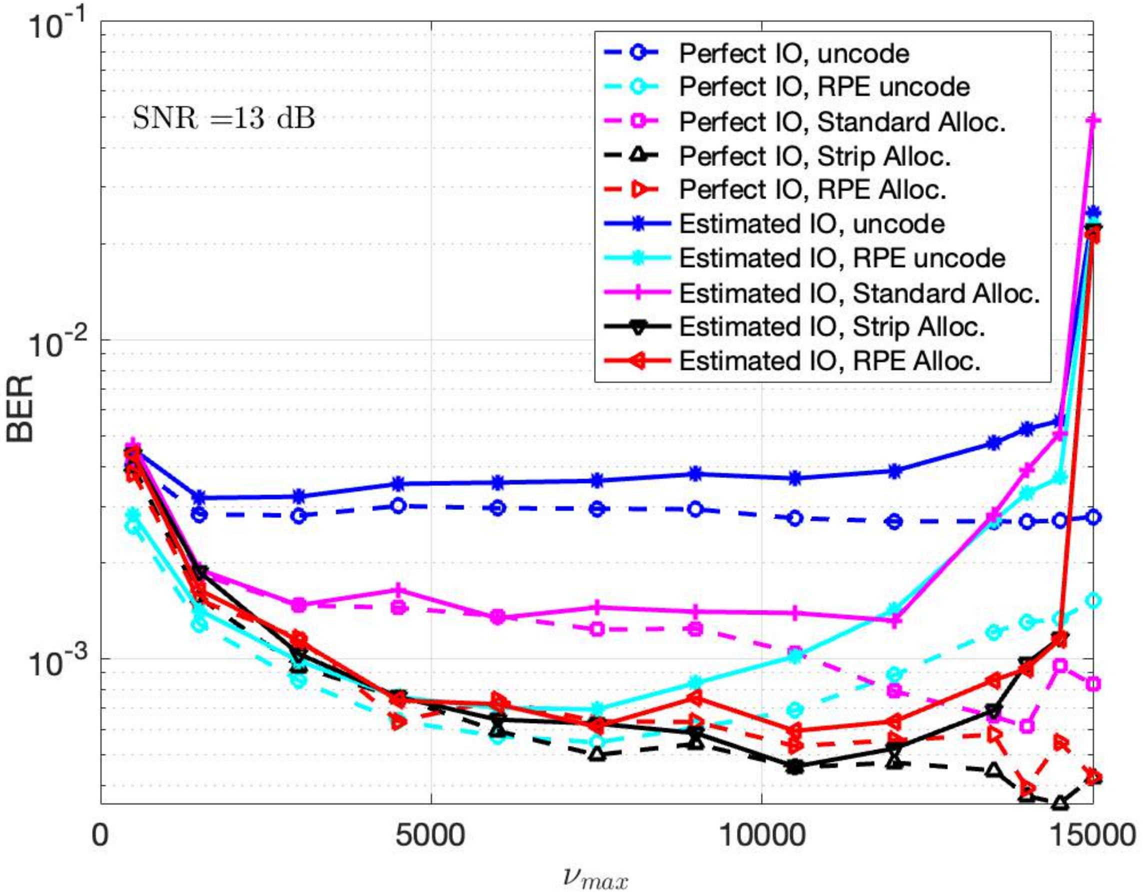
The Effect of Proposed Allocation Strategy on Uncoded Transmission

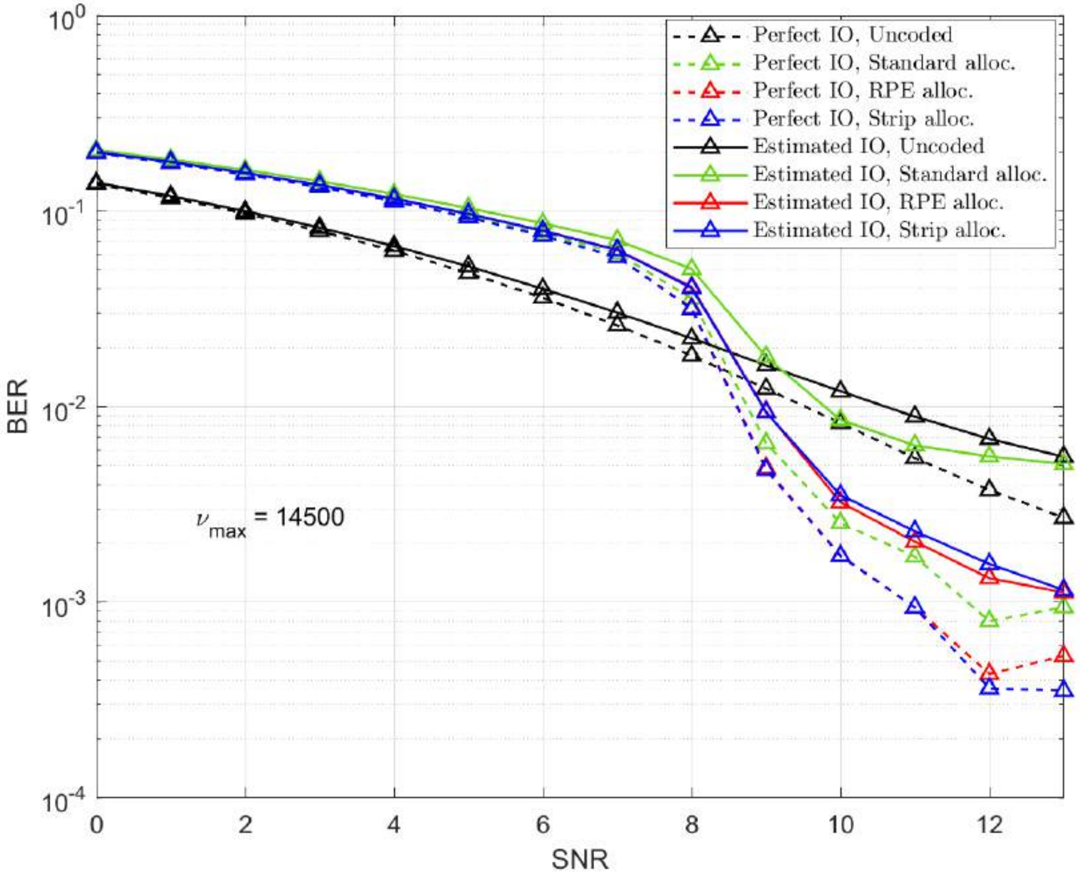


The Effect of Coding and Proposed Allocation Strategy on Coded Transmission

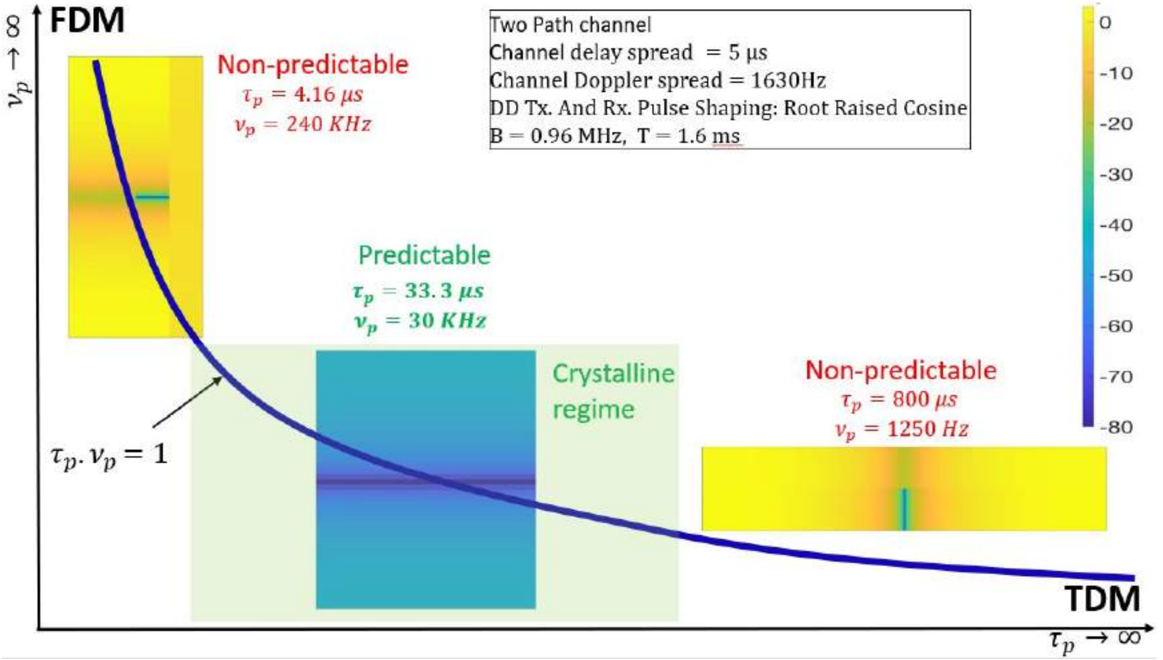


Comparison Between All Strategies for Varying Doppler Spread and Varying SNR

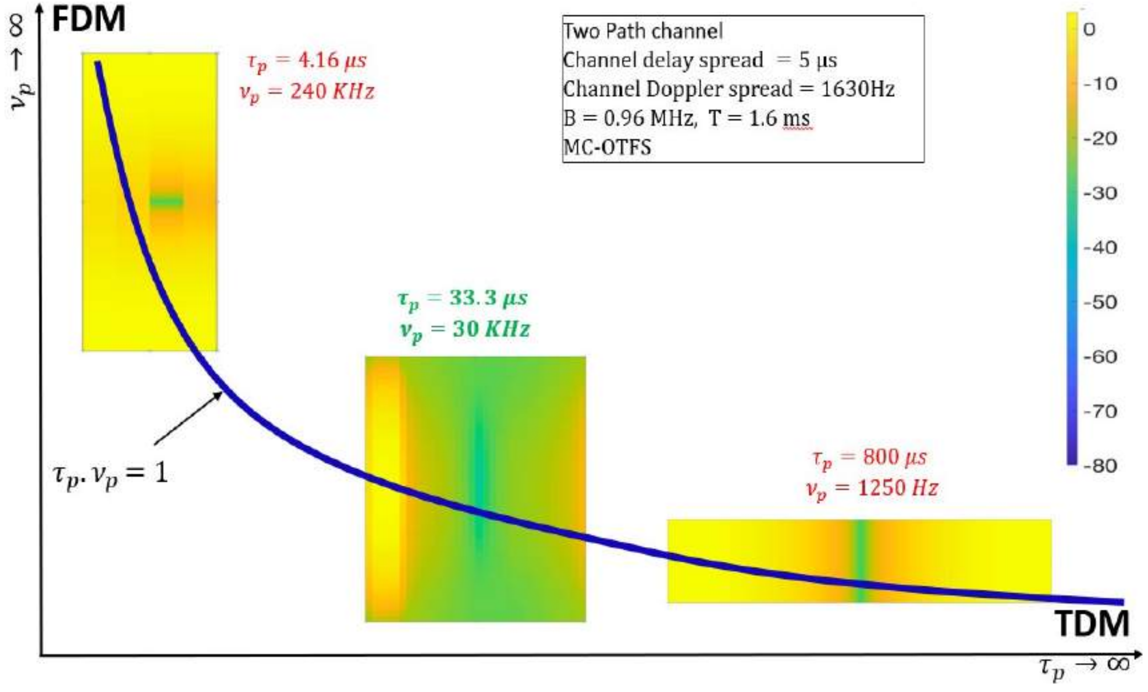




Zak-OTFS vs MC-OTFS Predictability of the I/O Relation



Zak-OTFS



MC-OTFS

LDPC Coding in MC-OTFS versus Zak-OTFS with Different Filters