Channel Estimation and Sensing for OTFS Modulation using 2D MUSIC

Akshay S. Bondre and Christ D. Richmond

Center of Excellence Meeting

March 27th 2023, Monday, 12pm EDT

Signals, Information, Inference, & Learning (SIIL) Group

Arizona State University

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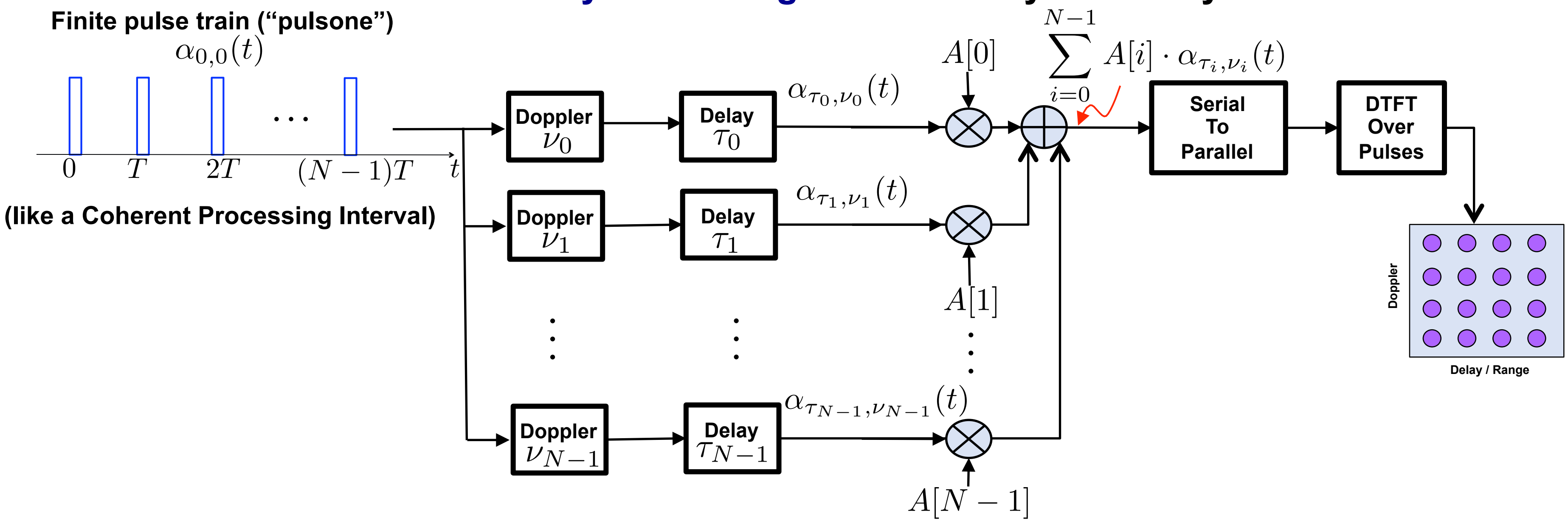
OTFS Modulation from Radar Perspective*

- **OTFS modulation transmits “synthetic targets” scaled by comm. symbols**



- **Doppler-Delay space is populated with communication symbols**
  - **Similar to pulse amplitude modulation, but in Doppler-Delay space**
- **Implement via pre- and post-processing block overlays to existing OFDM architecture****

*S.K. Mohammed, “Derivation of OTFS Modulation From First Principles”, *IEEE Trans. Veh. Tech.*, Aug 2021.
**R. Hadani, S. Rakib, S. Kons, M. Tsatsanis et al., “Orthogonal Time Frequency Space Modulation,” arXiv:1808.00519 [cs, math], Aug. 2018.

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Joint work with Nicolò Michelusi and Ahmed Alkhateeb.

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Outline

- **Introduction**
- MUSIC Method
- OTFS Channel Estimation via MUSIC
- 1D v/s 2D MUSIC
- Numerical Results
- Summary

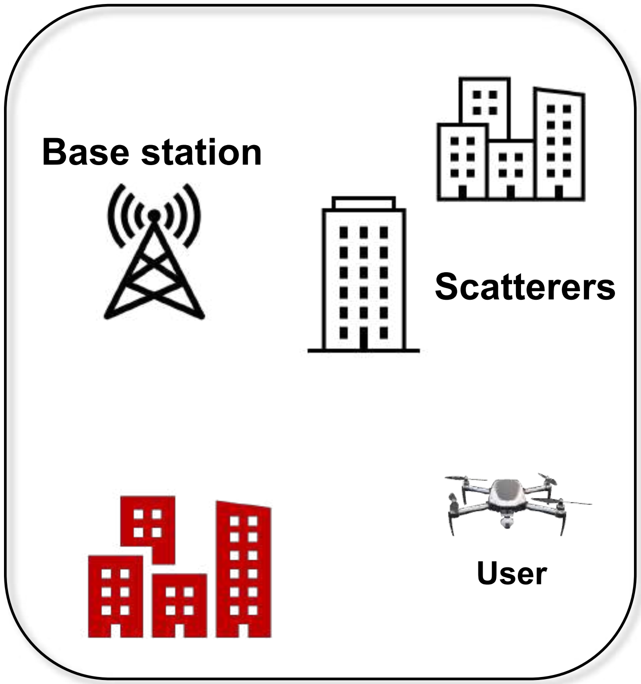
Dynamic Wireless Environments

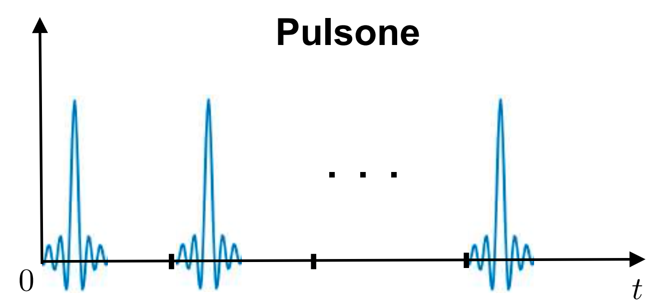
- **The wireless channel is characterized by:**
  - **Multipath scattering environment**
  - **User mobility**

- **The channel changes dynamically due to user mobility.**

- **Goal: Estimate and track the channel accurately**

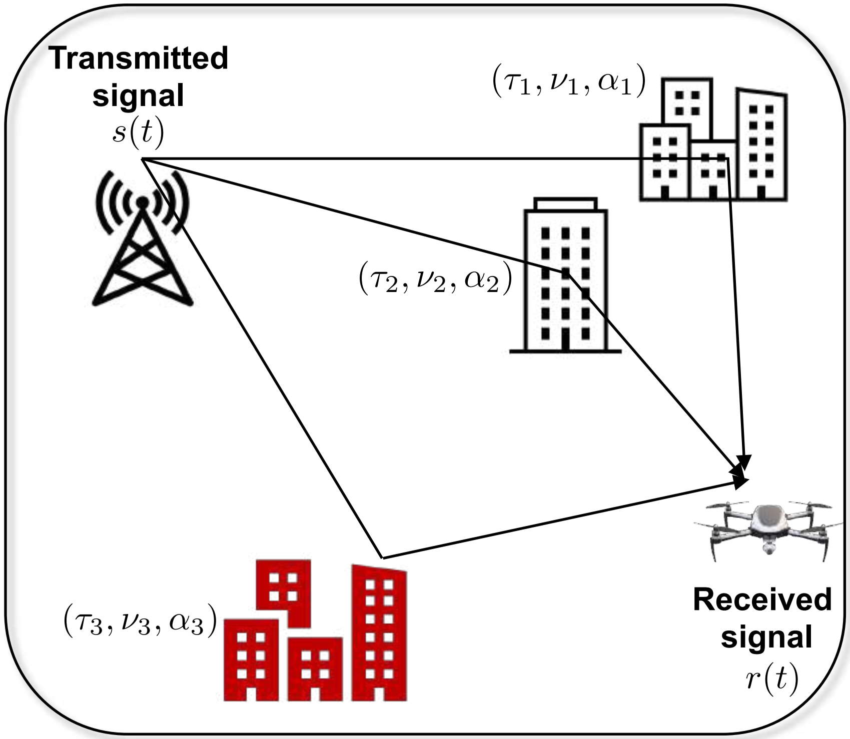
- **The OTFS waveform provides the ability to estimate and track the channel like a radar.**





Estimating Sparse Linear Time-Varying Channels

- D **multipath components parameterized by** $\{(\tau_i, \nu_i, \alpha_i)\}_{i=1}^{D}$
  - **Delays** τ_i**, Doppler shifts** ν_i
  - **Path gains** α_i



- $$r(t) = \sum_{i=1}^{D} \alpha_i s(t - \tau_i) e^{j2\pi\nu_i t}$$

- **Goal: Estimate** $\{(\tau_i, \nu_i, \alpha_i)\}_{i=1}^{D}$

- **Doppler Rayleigh resolution limit:**

$$\Delta\nu = \frac{1}{\text{Signal duration}}$$

Ex.: $\Delta\nu = 1/(1 \text{ ms}) = 1 \text{ kHz}$

- **Fractional delays and Dopplers** ⇒ **Need for parametric super-resolution methods**

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Line Spectral Estimation

- **Received Signal:** $y[n] = \sum_{i=1}^{D} \alpha_i e^{j\omega_i n} + w[n], \qquad n = 0, \ldots, N-1$
  $w[n] \sim \mathcal{CN}(0, \sigma^2)$

- **Goal: Estimate frequencies** $\{\omega_i\}_{i=1}^{D}$ **and complex gains** $\{\alpha_i\}_{i=1}^{D}$ **.**

- **Vector form:** $\boxed{\mathbf{y} = \sum_{i=1}^{D} \alpha_i \mathbf{a}(\omega_i) + \mathbf{w} = \mathbf{A}\boldsymbol{\alpha} + \mathbf{w}}$
  - $\mathbf{y} = \left[y[0], \ldots, y[N-1]\right]^T$, $\mathbf{w} = \left[w[0], \ldots, w[N-1]\right]^T \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$
  - $\mathbf{a}(\omega_i) = \left[e^{j\omega_i(0)}, e^{j\omega_i(1)}, \ldots, e^{j\omega_i(N-1)}\right]^T$, $\mathbf{A} = \left[\mathbf{a}(\omega_1), \ldots, \mathbf{a}(\omega_D)\right]$ **and** $\boldsymbol{\alpha} = \left[\alpha_1, \alpha_2, \ldots, \alpha_D\right]^T$**.**

- **Max. likelihood (ML) solution:** D **- dimensional search**
  - **Pros: Achieves Cramér-Rao bound asymptotically**
  - **Cons: Computationally expensive**

The MUSIC (Multiple Signal Classification) Method

- MUSIC only requires a single 1D search.

- $\mathbf{y} = \mathbf{A}\boldsymbol{\alpha} + \mathbf{w}$ ⇨ Covariance matrix $\boxed{\mathbf{K} = E\left[\mathbf{y}\mathbf{y}^H\right]}$

- $\{\lambda_i, \mathbf{v}_i\}_{i=1}^{N}$: Eigenvalues and eigenvectors of $\mathbf{K}$, $\lambda_1 \geq \lambda_2 \geq \cdots \geq \lambda_N$
  - Eigenvector matrix: $\mathbf{V} = [\underbrace{\mathbf{v}_1, \ldots, \mathbf{v}_D}_{\text{Spans the column space of } \mathbf{A}}, \underbrace{\mathbf{v}_{D+1}, \ldots, \mathbf{v}_N}_{\substack{\mathbf{G} \\ \text{Noise subspace matrix}}}]$

$\boxed{\text{Range}(\mathbf{A}) \perp \text{Range}(\mathbf{G})}$

- The true frequency values $\omega_i, i = 1, \ldots, D$ are the only solutions of

$$\boxed{\mathbf{a}^H(\omega_i)\mathbf{G}\mathbf{G}^H\mathbf{a}(\omega_i) = 0}$$

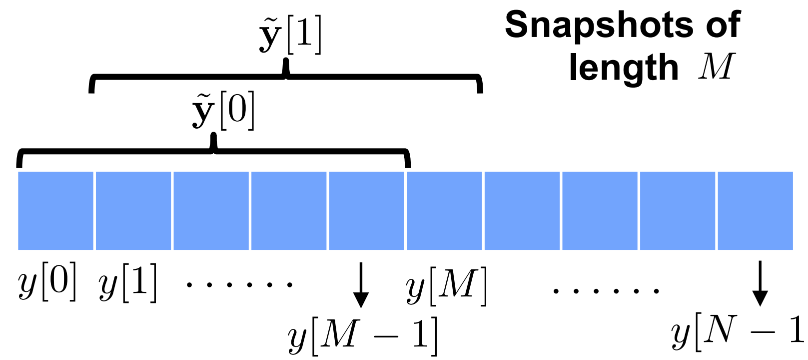
Knowledge of number of scatterers D required

P. Stoica and R. L. Moses, *Spectral analysis of signals*.

The MUSIC Method

- **Practically, the covariance matrix must be estimated from data**
  - **Single snapshot case: Consider subsequences of $\mathbf{y}$ as snapshots**

- **Let $\tilde{\mathbf{y}}(l), l = 0, \ldots, N_{\text{snap}} - 1$**
  - N_{snap} **snapshots of length** M



- **Sample covariance matrix**

$$\hat{\mathbf{K}} = \frac{1}{N_{\text{snap}}} \sum_{l=0}^{N_{\text{snap}}-1} \tilde{\mathbf{y}}(l)\tilde{\mathbf{y}}(l)^H$$ **, noise subspace matrix** $\hat{\mathrm{G}}$

- **Spectral MUSIC:** $\{\hat{\omega}_i\}_{i=1}^{D}$ **are the** D **peaks of the pseudospectrum**

$$P_{\text{MUSIC}}(\omega) = \frac{1}{\mathbf{a}^H(\omega)\hat{\mathbf{G}}\hat{\mathbf{G}}^H\mathbf{a}(\omega)}$$ ⇨ **1D search**

P. Stoica and R. L. Moses, *Spectral analysis of signals*.

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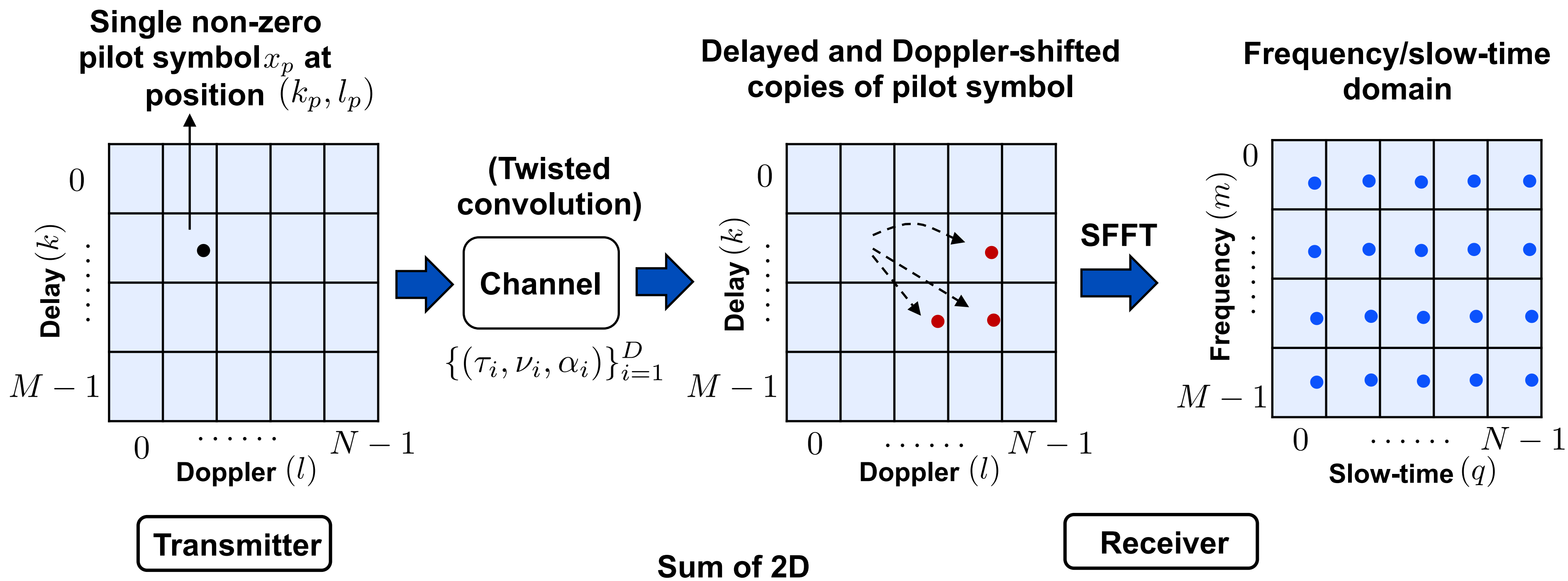
OTFS Signal Model

- $M \times N$ **OTFS delay-Doppler grid:**



Sum of 2D complex exponentials

x_p, k_p, l_p **are know pilot parameters**

$$R[m,q] = \frac{x_p}{\sqrt{MN}} \sum_{i=1}^{D} \alpha_i e^{-j2\pi\left(\frac{k_p}{M}+\frac{\tau_i}{T}\right)m} e^{j2\pi\left(\frac{l_p}{N}+\nu_i T\right)q}$$

$$m = 0, \dots, M-1$$
$$q = 0, \dots, N-1$$

2D MUSIC in the Frequency/Slow-time Domain

- **Frequency/slow-time domain signal:**

Noise term

$$W[m,q] \sim \mathcal{CN}(0,\sigma^2)$$

$$\underbrace{R[m,q]}_{[\mathbf{R}]_{m,q}} = \frac{x_p}{\sqrt{MN}} \sum_{i=1}^{D} \alpha_i \underbrace{e^{-j2\pi\left(\frac{k_p}{M}+\frac{\tau_i}{T}\right)m} e^{j2\pi\left(\frac{l_p}{N}+\nu_i T\right)q}}_{[\mathbf{B}(\tau,\nu)]_{m,q}} + \underbrace{W[m,q]}_{[\mathbf{W}]_{m,q}}$$

$$m = 0, \ldots, M-1$$
$$q = 0, \ldots, N-1$$

- **Matrix form:** $\mathbf{R} = \frac{x_p}{\sqrt{MN}} \sum_{i=1}^{D} \alpha_i \mathbf{B}(\tau_i,\nu_i) + \mathbf{W}$

- **Vectorizing,**

$$\mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$$

$$\boxed{\mathbf{y} = \mathrm{vec}(\mathbf{R}) = \frac{x_p}{\sqrt{MN}} \sum_{i=1}^{D} \alpha_i \mathbf{b}(\tau_i,\nu_i) + \mathbf{w}}$$

$$\mathbf{b}(\tau,\nu) = \mathbf{b}_1(\nu) \otimes \mathbf{b}_2(\tau) = \begin{bmatrix} 1 \\ e^{j2\pi\left(\frac{l_p}{N}+\nu T\right)} \\ \vdots \\ e^{j2\pi\left(\frac{l_p}{N}+\nu T\right)(N-1)} \end{bmatrix} \otimes \begin{bmatrix} 1 \\ e^{-j2\pi\left(\frac{k_p}{M}+\frac{\tau}{T}\right)} \\ \vdots \\ e^{-j2\pi\left(\frac{k_p}{M}+\frac{\tau}{T}\right)(M-1)} \end{bmatrix}$$

- **Thus, we can apply the MUSIC method for estimating $\{\tau_i,\nu_i\}_{i=1}^{D}$:**

$$\boxed{\mathbf{b}^H(\tau,\nu)\mathbf{G}\mathbf{G}^H\mathbf{b}(\tau,\nu) = 0}$$

2D Root-MUSIC

- **2D spectral-MUSIC:**
  - **2D search over (τ,ν) grid for finding D peaks of:**

$$\boxed{\mathbf{b}^H(\tau,\nu)\mathbf{E}_n\mathbf{b}(\tau,\nu)=0}$$

$$\mathbf{G}\mathbf{G}^H=\mathbf{E}_n$$

$$P_{\text{MUSIC}}(\tau,\nu)=\frac{1}{\mathbf{b}^H(\tau,\nu)\mathbf{E}_n\mathbf{b}(\tau,\nu)}$$

- **2D root-MUSIC: Let $z_1=e^{j2\pi\left(\frac{l_p}{N}+\nu T\right)}$ and $z_2=e^{-j2\pi\left(\frac{k_p}{M}+\frac{\tau}{T}\right)}$**

$$\mathbf{b}(\tau,\nu)=\tilde{\mathbf{b}}(z_2,z_1)=\tilde{\mathbf{b}}_1(z_1)\otimes\tilde{\mathbf{b}}_2(z_2)$$, **where** $\tilde{\mathbf{b}}_1(z_1)=\begin{bmatrix}1\\ z_1\\ \vdots\\ z_1^{N-1}\end{bmatrix}$ $\tilde{\mathbf{b}}_2(z_2)=\begin{bmatrix}1\\ z_2\\ \vdots\\ z_2^{M-1}\end{bmatrix}$

$$\mathbf{b}^H(\tau,\nu)\mathbf{E}_n\mathbf{b}(\tau,\nu)=\tilde{\mathbf{b}}^T(z_2^{-1},z_1^{-1})\mathbf{E}_n\tilde{\mathbf{b}}(z_2,z_1)=0 \Rightarrow$$ **Polynomial in two variables** (z_1,z_2)

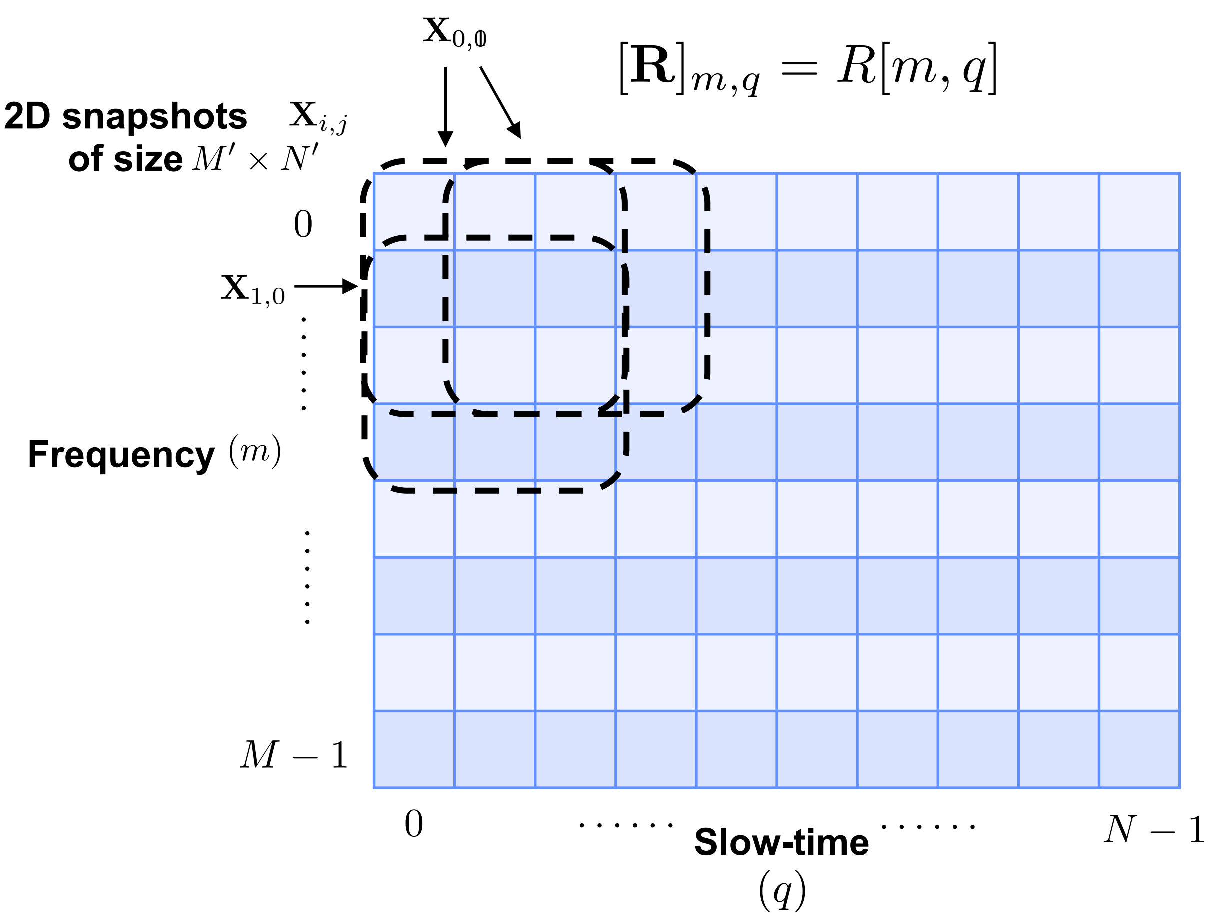
- **Solutions for (z_1,z_2) can be obtained by solving two separate polynomial rooting problems*.**
  - **2D search not required $\Rightarrow$ Computationally feasible**

(Consequence of the complex exponential steering vector structure)

*J. Lee, J. Park, and J. Chun, "Weighted Two-Dimensional Root MUSIC for Joint Angle-Doppler Estimation With MIMO Radar," *IEEE Trans. on Aerosp. and Elect. Syst.*, vol. 55, no. 3, pp. 1474–1482, Jun. 2019..

Snapshots for Covariance Estimation

- **2D subsequences are considered as snapshots**



- **Consecutive snapshots shifted by one sample**

Vectorized version

- $\mathbf{y} = \text{vec}(\mathbf{R}) = \frac{x_p}{\sqrt{MN}} \sum_{i=1}^{D} \alpha_i \mathbf{b}(\tau_i, \nu_i) + \mathbf{w}$
- **Snapshots:** $\mathbf{x}_{i,j} = \text{vec}(\mathbf{X}_{i,j})$

  $i = 0, \ldots, M - M', j = 0, \ldots, N - N'$

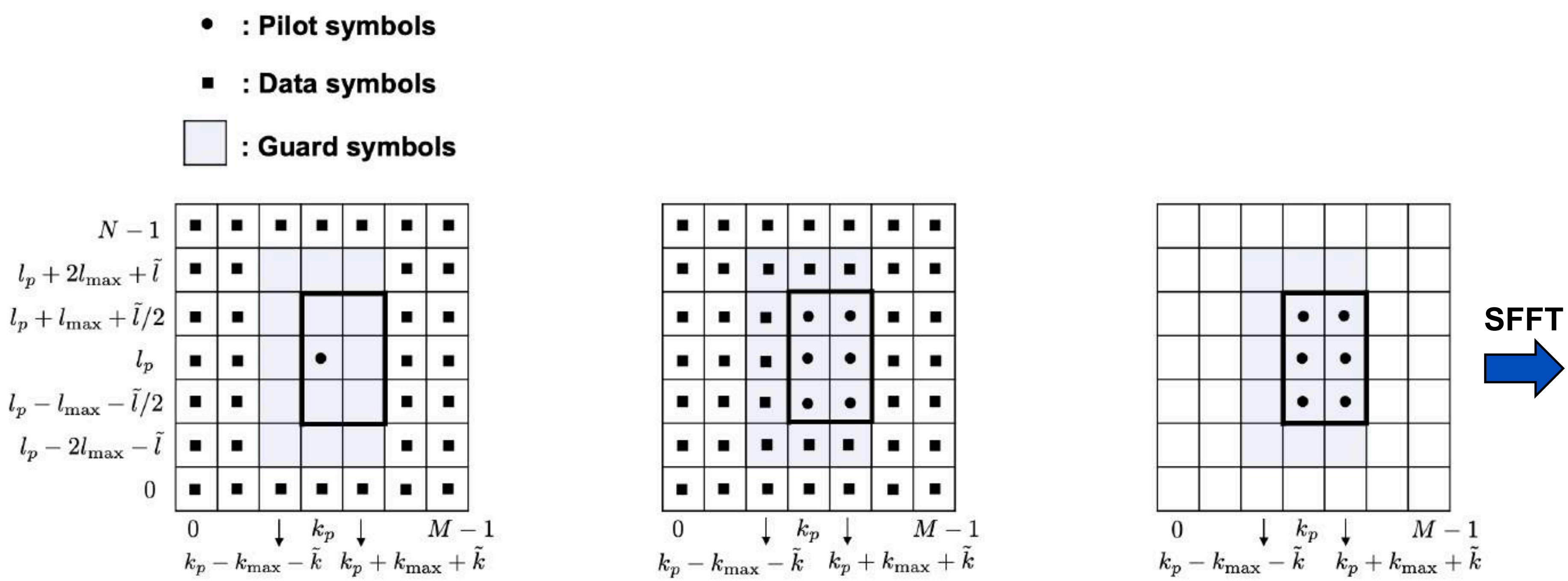
  $N_{\text{snap}} = (M - M' + 1)(N - N' + 1)$

- **Sample covariance matrix**

$$\hat{\mathbf{K}} = \frac{1}{N_{\text{snap}}} \sum_{i=0}^{M-M'} \sum_{j=1}^{N-N'} \mathbf{x}_{i,j} \mathbf{x}_{i,j}^H$$

2D MUSIC for Embedded Pilot Structure

- **Pilot symbol separated from data symbols with a guard interval**
  - **Max. Doppler tap** $l_{\max} = \nu_{\max} T_w / 2$ **, Max. delay tap** $k_{\max} = \tau_{\max} B$
  - $\tilde{k}$ **and** $\tilde{l}$ **chosen to control interference between data and pilot symbols**



Transmitted OTFS frame

Received OTFS frame

Filtered OTFS frame used for 2D MUSIC algorithm

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Cramér-Rao Bound (CRB) for Delay-Doppler Estimation

- **Unknown parameter vector:** $\boldsymbol{\theta} = [\theta_1, \ldots, \theta_P]^T$
- **Received signal having the form:** $\mathbf{y} = \boldsymbol{\mu}(\boldsymbol{\theta}) + \mathbf{w}$ $\qquad \mathbf{w} \sim \mathcal{CN}(\mathbf{0}, \sigma^2\mathbf{I})$
- **Fisher information matrix (FIM)** $\mathbf{J}(\boldsymbol{\theta})$**:**

$$[\mathbf{J}(\boldsymbol{\theta})]_{i,k} = \frac{2}{\sigma^2} \cdot \mathrm{Re}\left\{ \left(\frac{\partial \boldsymbol{\mu}}{\partial \theta_i} \right)^H \left(\frac{\partial \boldsymbol{\mu}}{\partial \theta_k} \right) \right\}$$

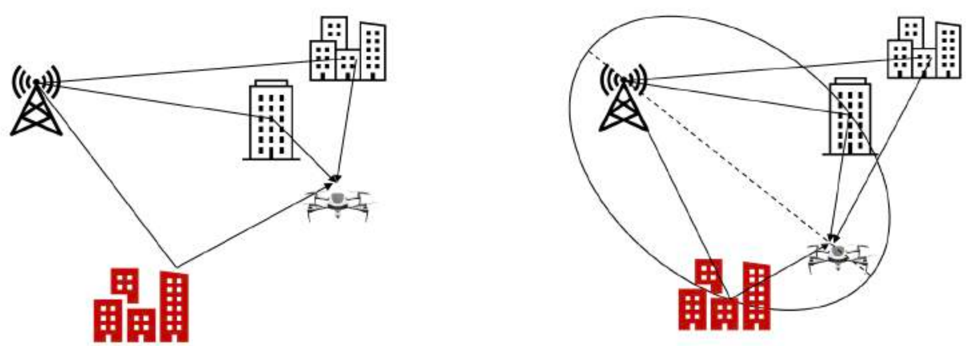
$$\mathbf{y} = \frac{x_p}{\sqrt{MN}} \sum_{i=1}^{D} \alpha_i \mathbf{b}(\tau_i, \nu_i) + \mathbf{w}$$

- **CRB matrix** $= \mathbf{J}^{-1}(\boldsymbol{\theta})$
  - **Diagonal elements of CRB provide a lower bound on variance of any unbiased estimator** $\Rightarrow$ $\mathbb{E}[(\hat{\theta}_i - \theta_i)^2] \geq \left[\mathbf{J}^{-1}(\boldsymbol{\theta})\right]_{i,i}$

- **Non-diagonal elements of FIM** $\Rightarrow$ **Possible coupling between unknown parameters (Motivation for joint estimation)**

1D v/s 2D Estimation

- **Turns out the coupling between delays and Dopplers is negligible***
- **Delays and Dopplers can be estimated individually, but pairing procedures are necessary to find the correct delay-Doppler pair**
  - **2D MUSIC pairs delays and Dopplers automatically**
- **Estimating Dopplers for a specific delay requires the knowledge of the number of Dopplers with that particular delay**



Two scatterers with same delay and different Doppler shift

Frequent estimation of model order required as the channel changes due to user mobility

*Dogandzic and A. Nehorai, "Cramer-Rao bounds for estimating range, velocity, and direction with an active array," *IEEE Trans. on Sig. Proc.*, vol. 49, no. 6, pp. 1122–1137, 2001.

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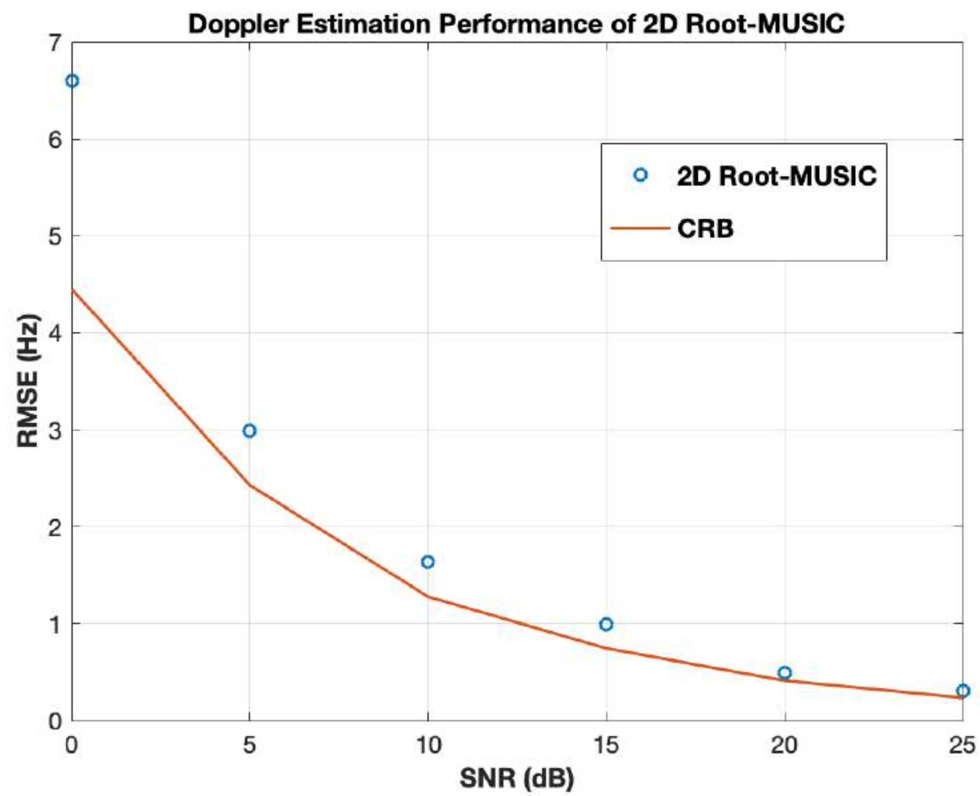
Numerical Results

- **Simulation parameters:**
  - $M = N = 64$
  - $B = 2\ \mathrm{MHz}$
  - $D = 4$ **paths**
  - $\tau_{\max} = 10T_s$
  - $\nu_{\max} = 3000\ \mathrm{Hz}$
  - $\tau_i \sim \mathcal{U}(0, \tau_{\max})$
  - $\nu_i \sim \mathcal{U}(-\nu_{\max}, \nu_{\max})$
  - $h_i \sim \mathcal{CN}(0, 1)$

- **2D Root MUSIC method*, no 2D search involved.**



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Summary

- Estimating delays and Doppler shifts reduces to a 2D sinusoidal frequency estimation problem in the frequency/slow-time domain.
- Computationally feasible 2D root-MUSIC can be applied.
- 2D MUSIC can be applied to an embedded pilot OTFS frame by filtering out the data symbols.
- 2D MUSIC automatically gives paired delay-Doppler estimates and requires less frequent model order estimation.
- Future work:
  - Exploring more general OTFS pilot sequences and the application of 2D root-MUSIC-type methods to such pilot sequences

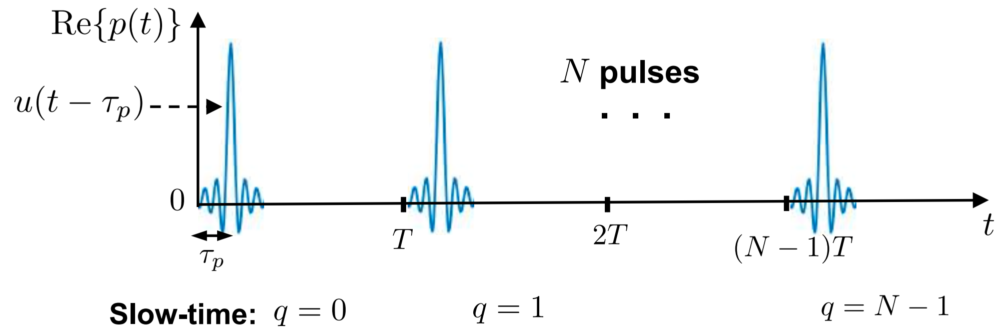
Thank You

Backup Slides

OTFS Received Signal Model

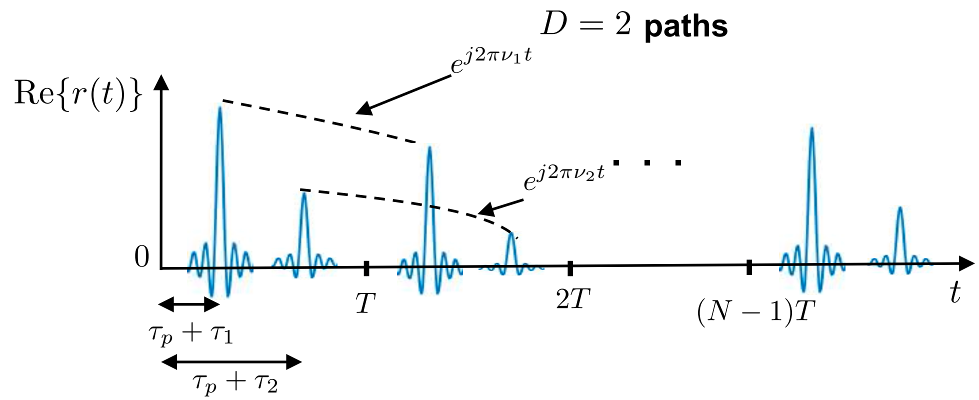
- **Transmitted pilot signal:**

$$p(t) = x_p \sum_{q=0}^{N-1} u(t - \tau_p - qT) e^{j2\pi\nu_p qT}$$



- **Received signal:**

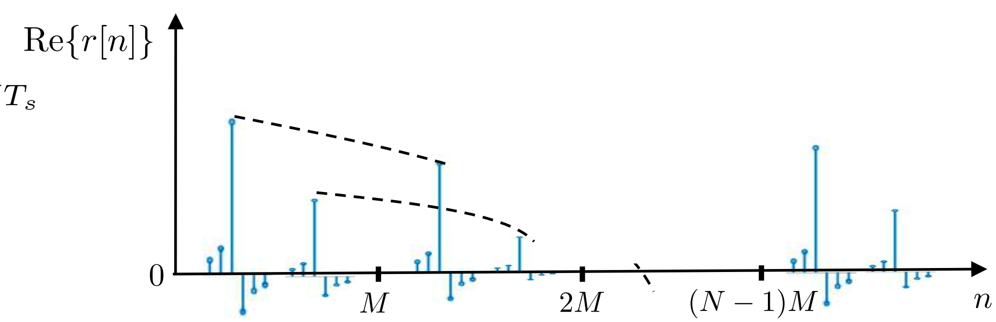
$$r(t) = \sum_{i=1}^{D} \alpha_i p(t - \tau_i) e^{j2\pi\nu_i t}$$



- **Sampling at $T_s = 1/B$,**

$$r[n] = r(nT_s) \approx \sum_{i=1}^{D} \alpha_i p(nT_s - \tau_i) e^{j2\pi\nu_i \lfloor \frac{n}{M} \rfloor M T_s}$$

$$n = 0, \ldots, MN-1$$

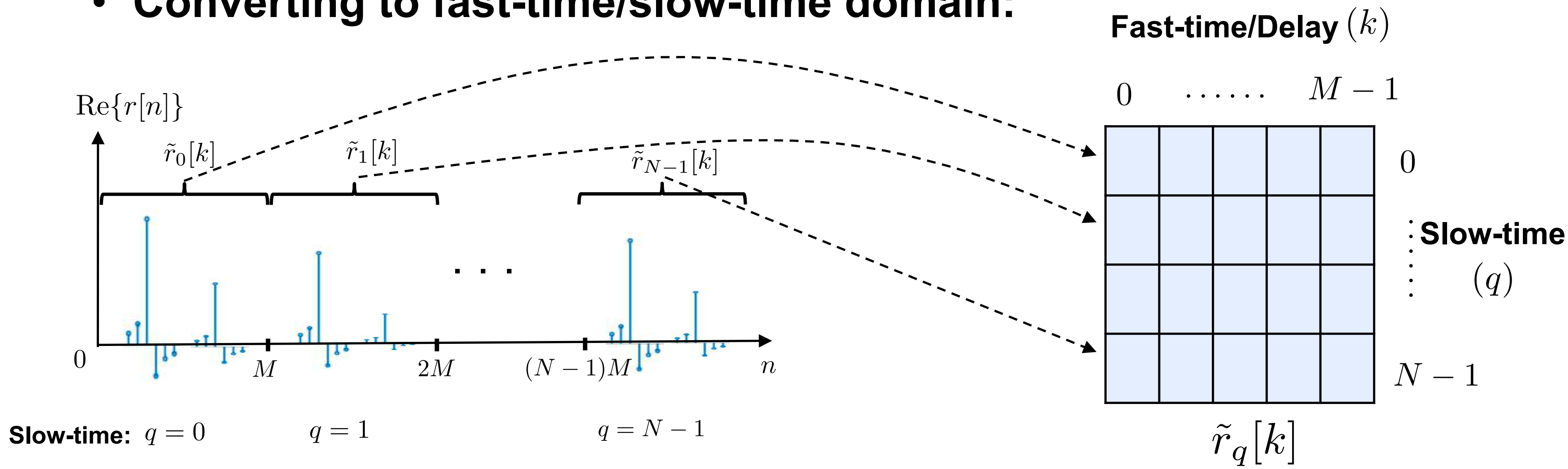


Fast-time/Slow-time Domain

- **Received signal:**

$$r[n] = r(nT_s) \approx \sum_{i=1}^{D} \alpha_i p(nT_s - \tau_i) e^{j2\pi\nu_i \lfloor \frac{n}{M} \rfloor MT_s} \qquad n = 0, \ldots, MN-1$$

- **Converting to fast-time/slow-time domain:**



$$\tilde{r}_q[k] \approx x_p \sum_{i=1}^{D} \alpha_i u(kT_s - \tau_p - \tau_i) e^{j2\pi(\nu_p + \nu_i)qT} \qquad \begin{array}{l} k = 0, \ldots, M-1 \\ q = 0, \ldots, N-1 \end{array}$$

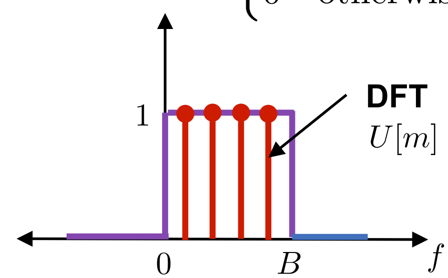
Frequency/Slow-time Domain

$$\tilde{r}_q[k] = x_p \sum_{i=1}^{D} \alpha_i u(kT_s - \tau_p - \tau_i) e^{j2\pi\nu_p qT} e^{j2\pi\nu_i qT} \qquad \begin{aligned} k &= 0, \ldots, M-1 \\ q &= 0, \ldots, N-1 \end{aligned}$$

- **DFT of fast-time/slow-time matrix along fast-time (delay):**

$$\tilde{r}_q[k] \underset{k}{\overset{\textbf{DFT}}{\longleftrightarrow}} R[m,q] = \sum_{k=0}^{M-1} \tilde{r}_q[k] e^{-j2\pi km/M}$$

$$u(t) \longleftrightarrow U(f) = \begin{cases} 1 & f \in [0, B] \\ 0 & \text{otherwise.} \end{cases}$$



  - **Frequency variable** $m = 0, \ldots, M-1$
  - $R[m,q]$: **Frequency/slow-time domain**

- **Using the time-shift property of the DFT,**

$$u[k] = u(kT_s) \overset{\textbf{DFT}}{\longleftrightarrow} U[m] = 1 \qquad m = 0, \ldots, M-1$$

$$R[m,q] = x_p \sum_{i=1}^{D} \alpha_i e^{-j2\pi \frac{(\tau_p + \tau_i)m}{T}} e^{j2\pi(\nu_p + \nu_i)qT} \qquad \begin{aligned} m &= 0, \ldots, M-1 \\ q &= 0, \ldots, N-1 \end{aligned}$$