Learning in the Delay-Doppler Domain

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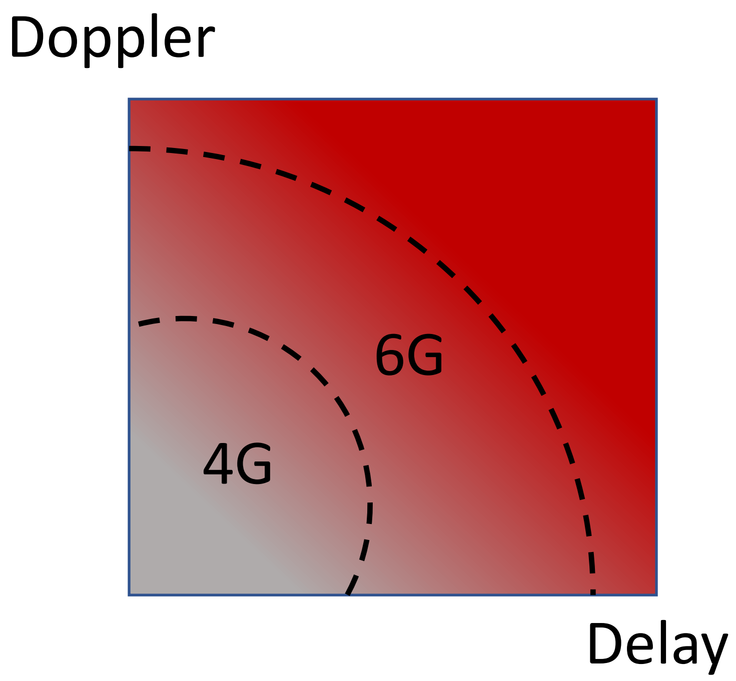
Abstract: We describe how pulsones interpolate between TDM and FDM, and when it is possible to learn input-output relations without learning the channel, opening the door to machine learning.

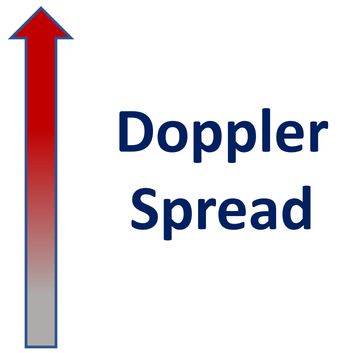
Learn More - IEEE BITS Magazine: *A Mathematical Foundation for Communications and Sensing in the Delay-Doppler Domain, Parts I and II* – in collaboration with Saif Khan Mohammed, Ronny Hadani, and Ananthanarayanan Chockalingam

Disclosure: Advisor to Cohere Technologies

So Many Channels, So Little Time



Leo-Satellite Channel
UAV/Aeronautical Channel
mmWave Mobile Channel
Terrestrial Mobile Channel
Terrestrial Pedestrian Channel



Today we design wireless systems using mathematical models

This approach is losing ground as wireless channels become more complex and Doppler becomes more significant

Might it be possible to operate model-free

We have Asked This Question Before

Newton’s Laws of Motion

Model-based approach that develops understanding at the most fundamental level.

Kepler’s Laws of Planetary Motion

Model-free approach that uses data to make predictions

Why Ask It Now?

Machine learning has revolutionized image and natural language processing

Data-driven discovery has revolutionized bioinformatics

Machine learning (ML) is about approximating functions – broad impact comes from the fact that it is particularly effective in high dimensions

Classically we measure complexity of functions by smoothness – how many times the function can be differentiated

ML measures complexity by how well the function can be approximated by a particular neural network model – reproducing kernel Hilbert spaces, for example

**Weinan E*, The Dawning of a New Era in Applied Mathematics, Notices of the American Mathematical Society, May 2021

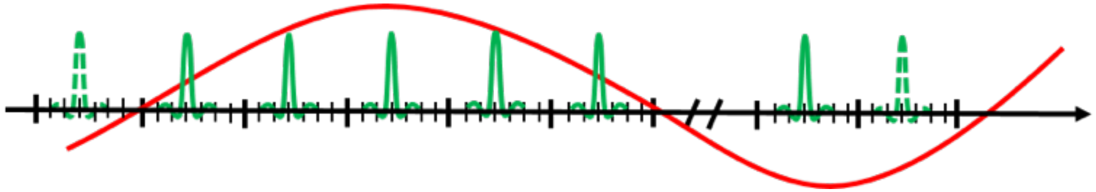
Localization in Delay and Doppler

Radar as a game of 20 questions with an operator

P.M. Woodward: *Probability and Information Theory, with Applications to Radar,* Pergamon Press, 1953

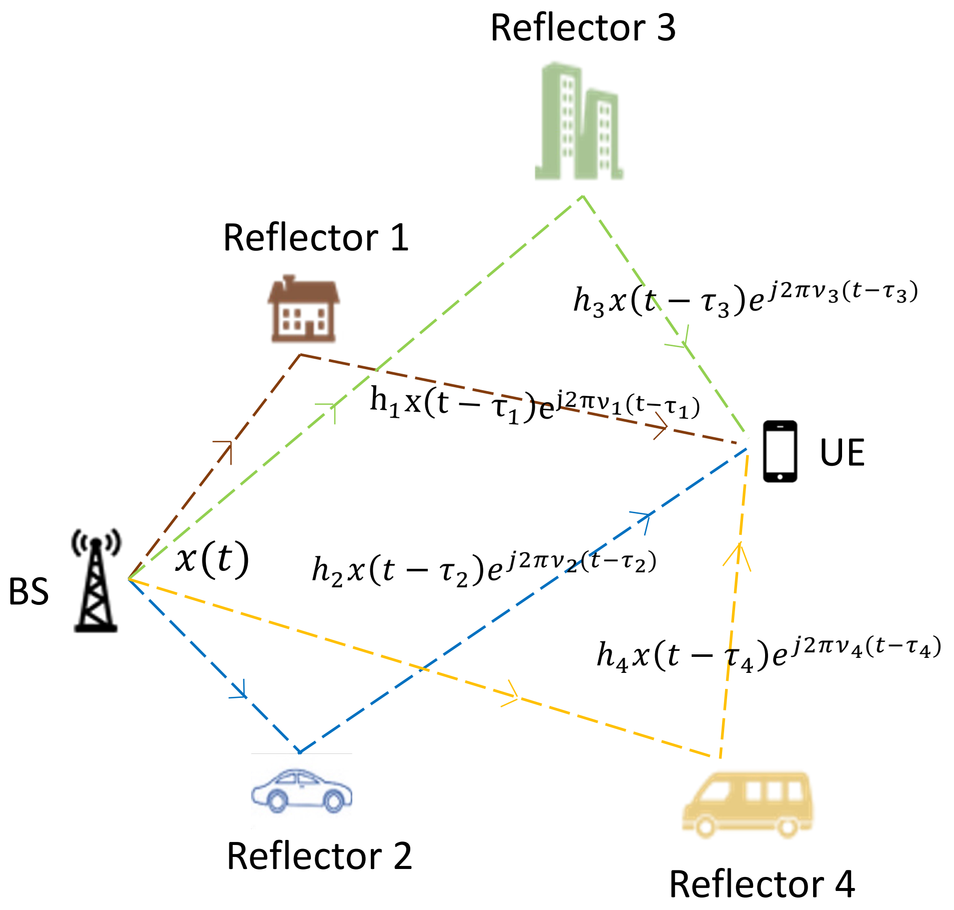
He viewed the problem of localizing a scatterer in delay and Doppler as using a waveform to ask questions of the operator defined by the radar scene

How to Design a Question:

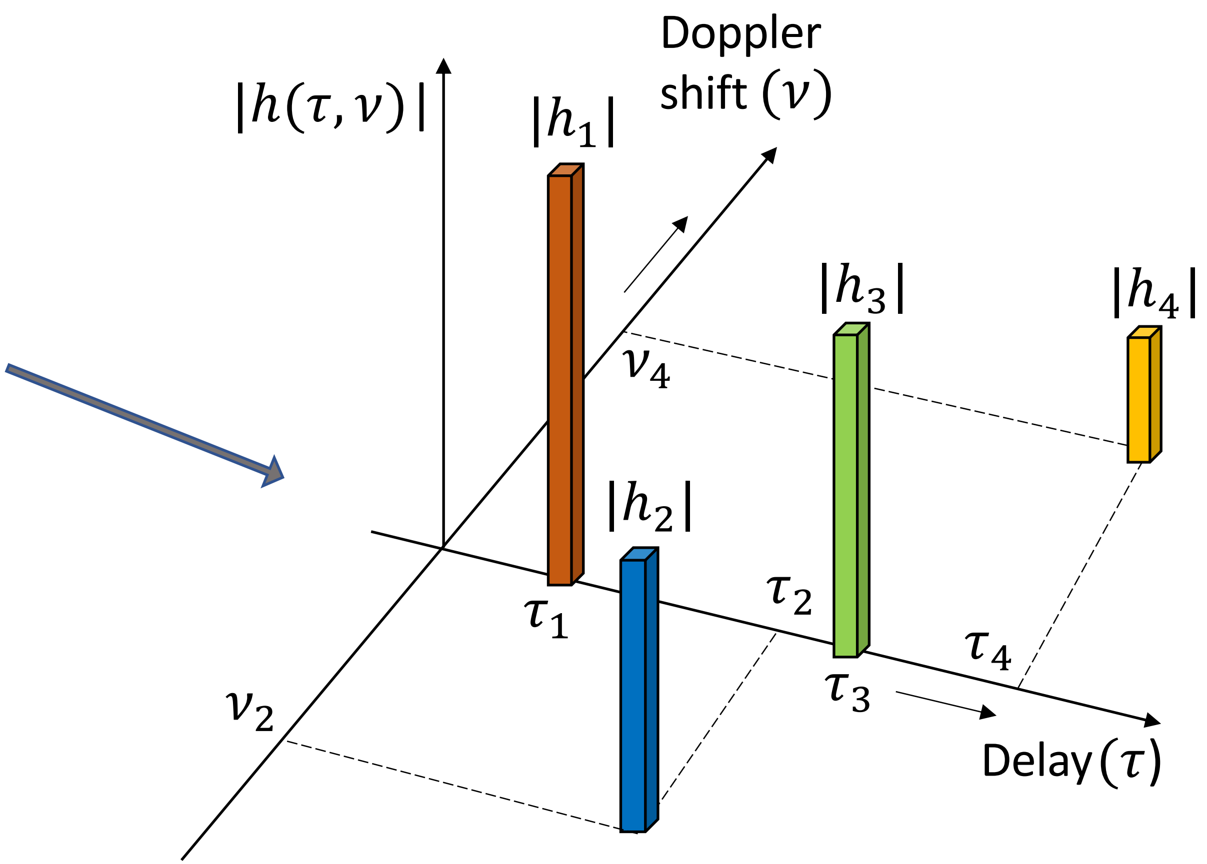


Prediction as a game of 20 questions with a doubly spread channel

Representing Doubly Spread Channels



Taps in Delay and Doppler

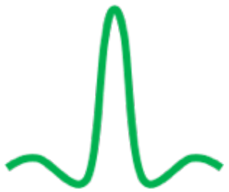


What is the right question?

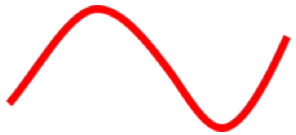
What Constitutes a Good Question?

Doubly Spread Channel: A sum of operators $D(\tau_i, \nu_j)$ introducing path delay τ_i and Doppler shift ν_j

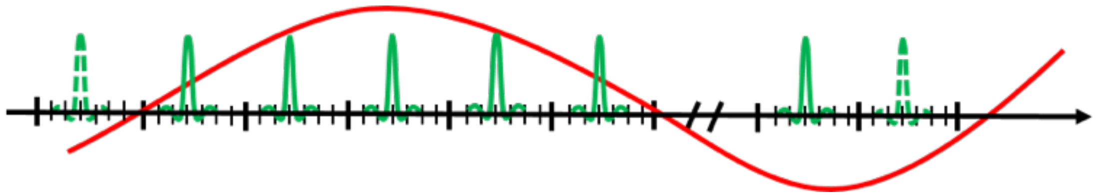
Waveforms are questions, returns are answers, objective is prediction



Time Domain (TD) Pulse: Good question for pure delay channels



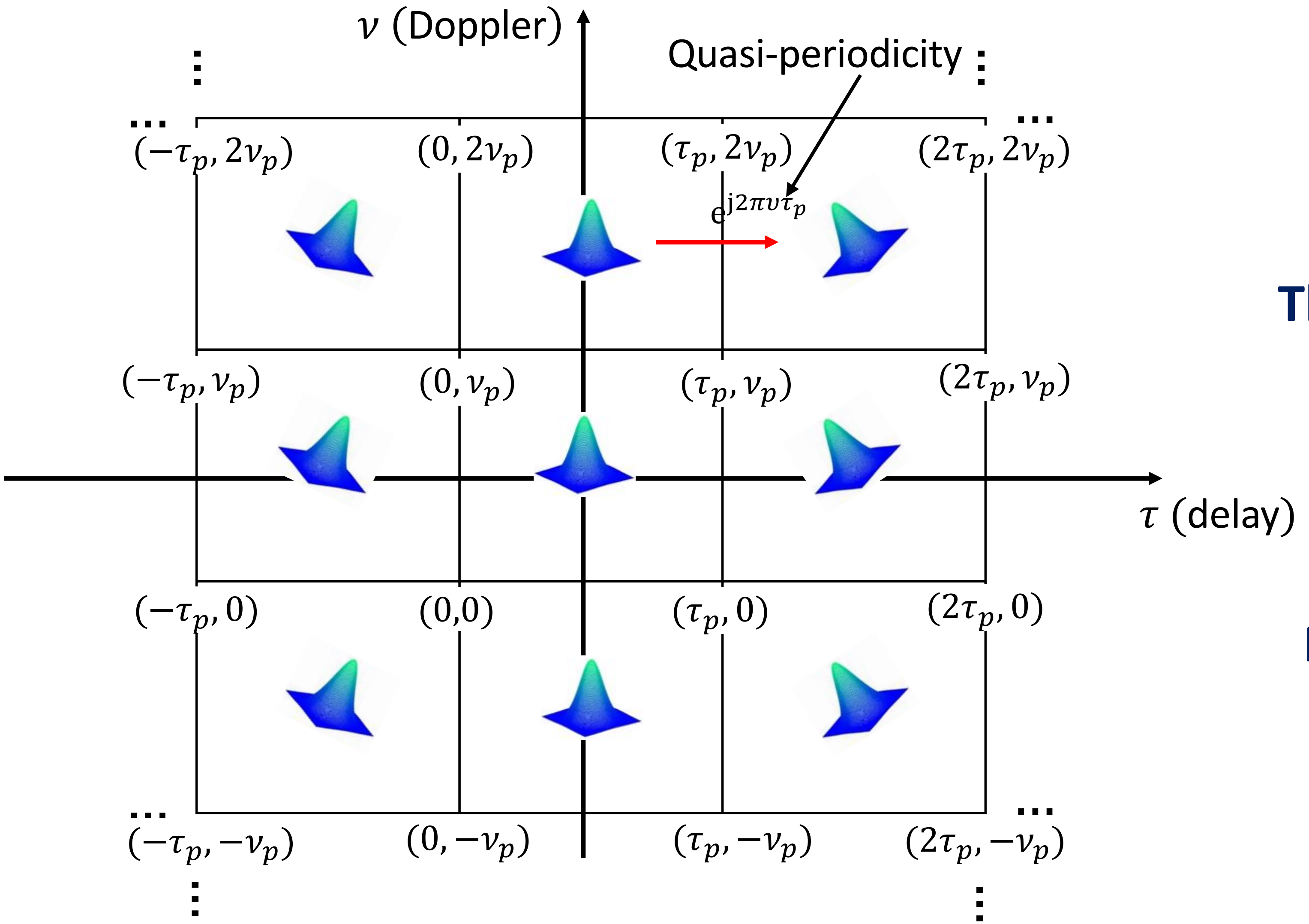
Frequency Domain (FD) Pulse: Good question for pure Doppler channels



Delay-Doppler (DD) Domain Pulse:

Good question for doubly spread channels

A Pulse in the Delay-Doppler Domain

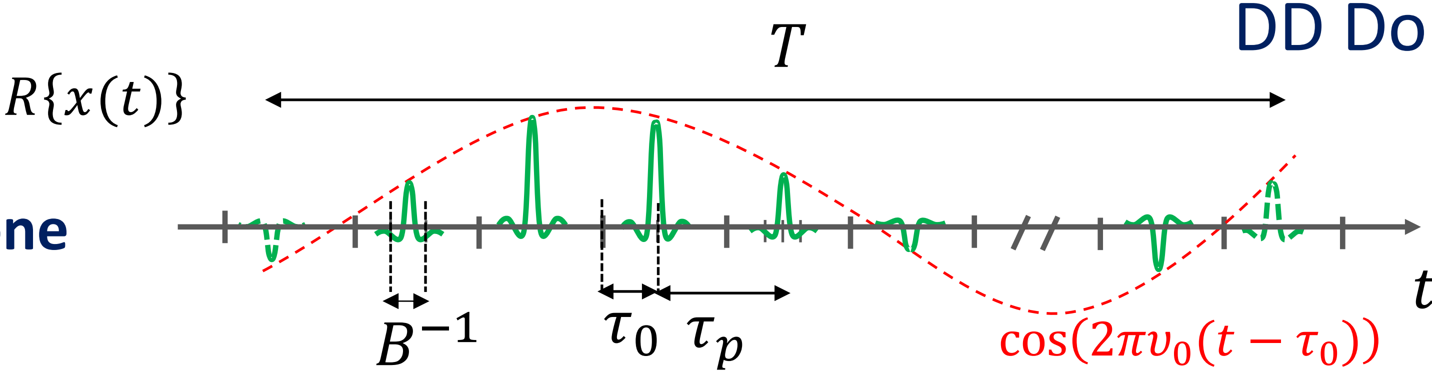


The DD realization of a TD signal is a quasi-periodic function

Fundamental Domain defined by the delay period τ_p and the Doppler period ν_p

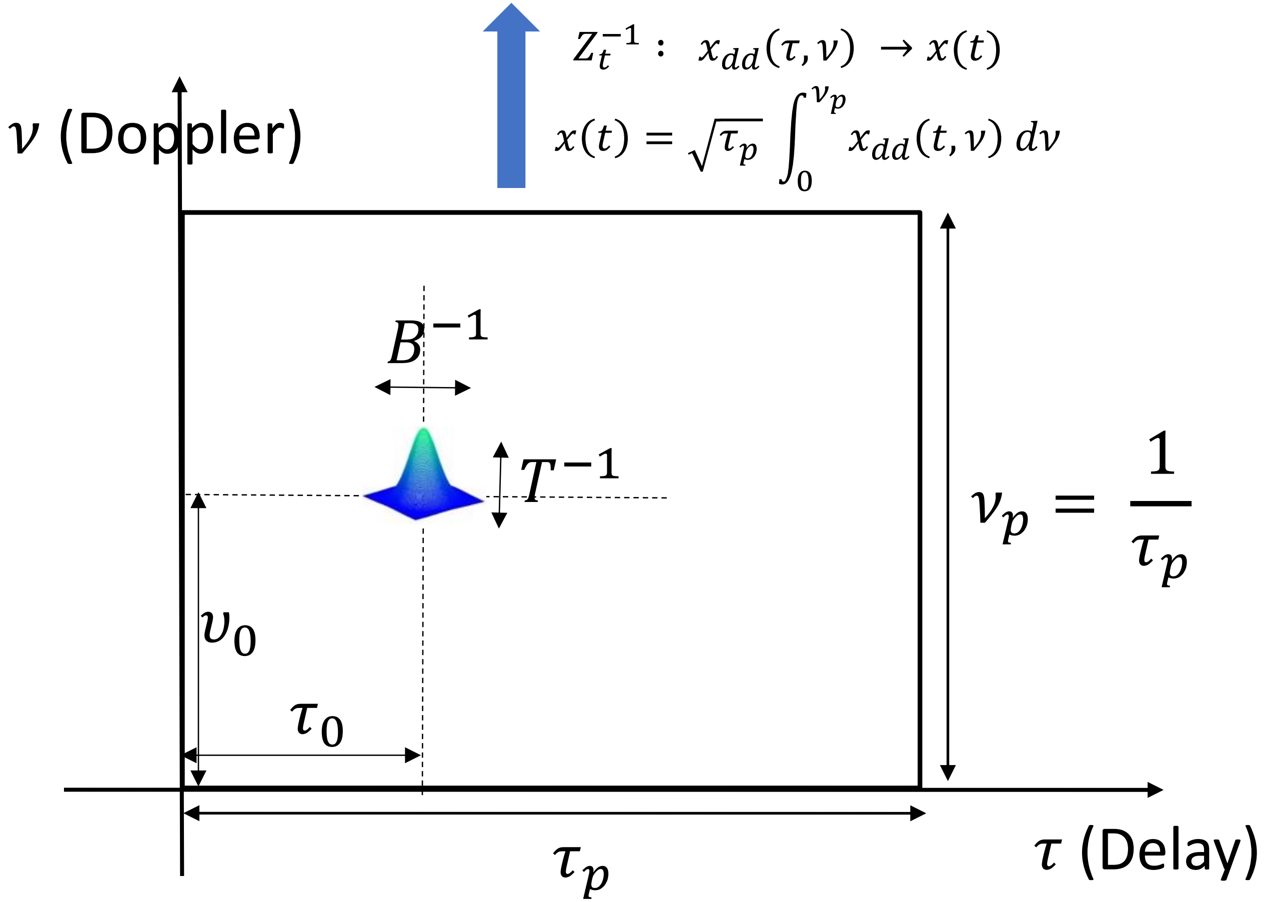
TD Pulsone from a Quasi-Periodic DD Domain Pulse

TD Pulsone



TDM is a limiting case

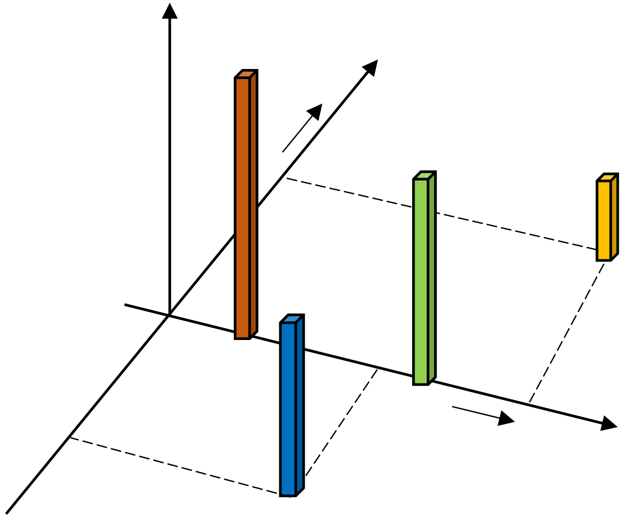
FDM is a limiting case

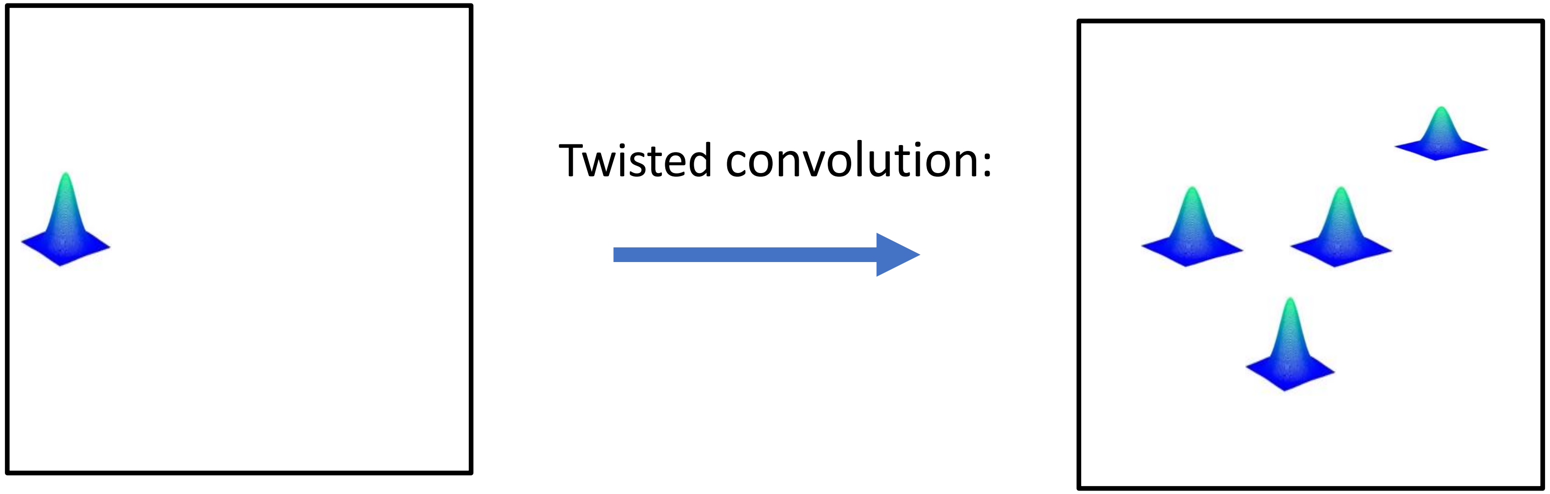


Pulsones parametrized by τ_p interpolate between TDM and FDM

When does a TD Pulsone question have predictive value?

Doubly Spread Channels Acting on Pulsones



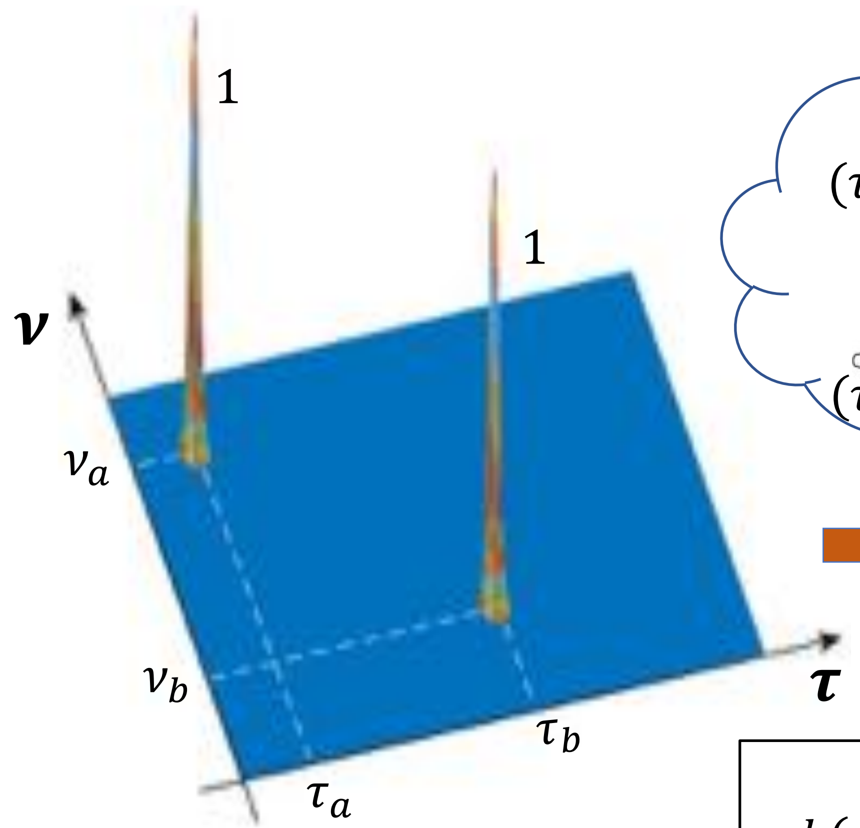


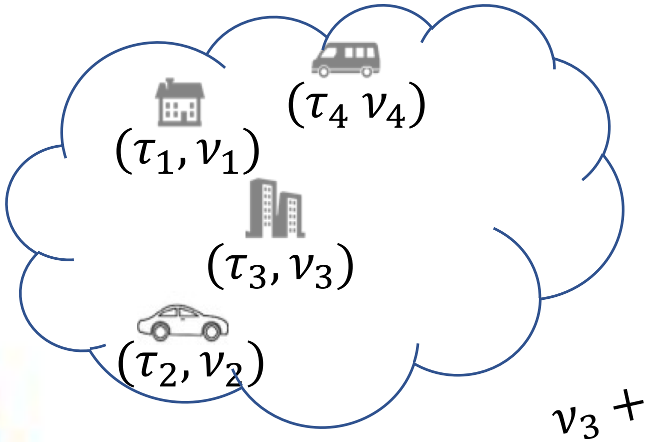
The interaction of a DD domain pulse with a doubly spread channel is predictable, as long as the delay domain period τ_p is greater than the channel path delay spread, and the Doppler domain period ν_p is greater than the path Doppler spread:

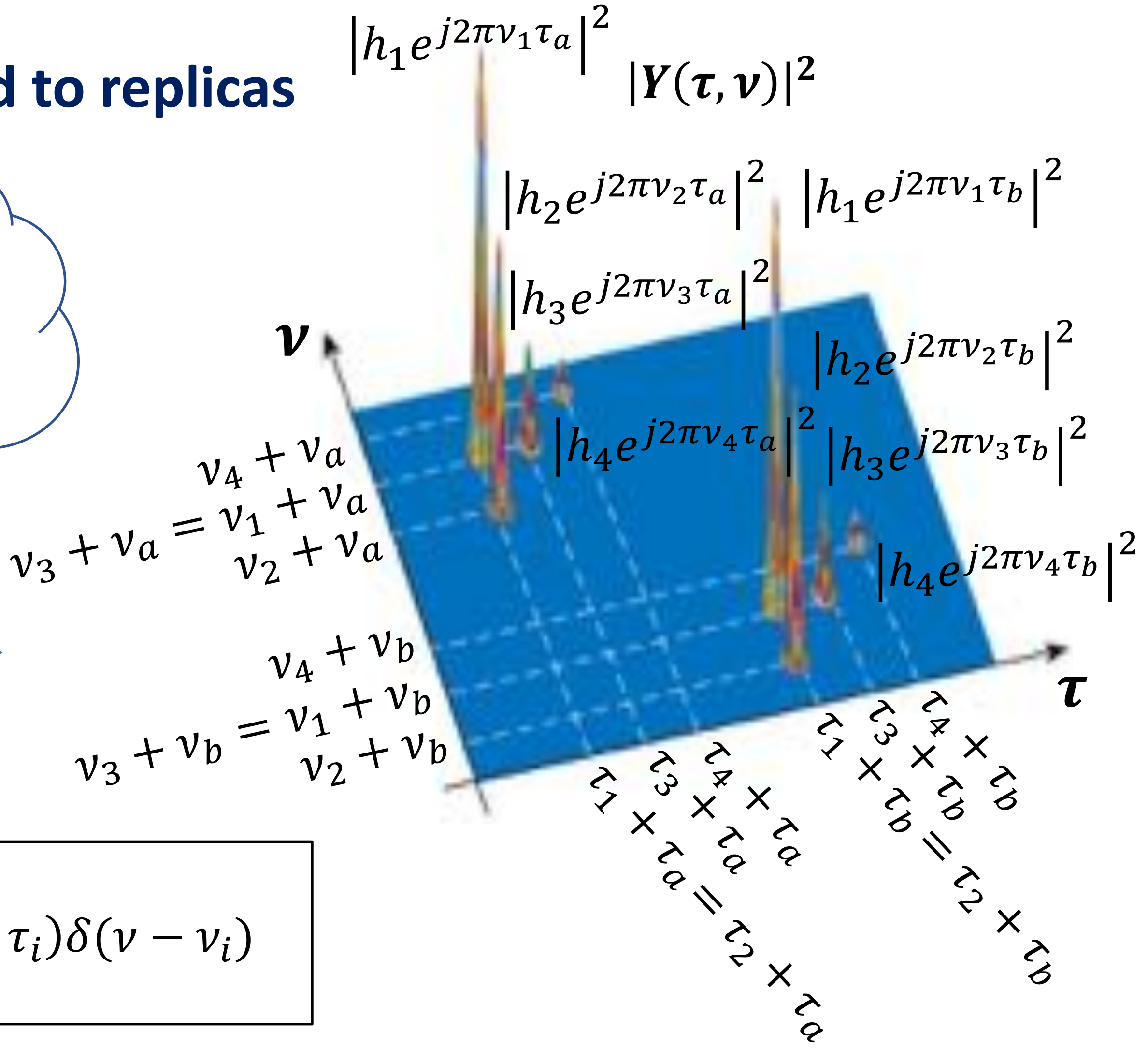
$$\tau_p > \max \tau_i - \min \tau_i \quad \text{and} \quad \nu_p > \max \nu_i - \min \nu_i$$

The Meaning of Predictability

Wireless channel dynamics lead to replicas





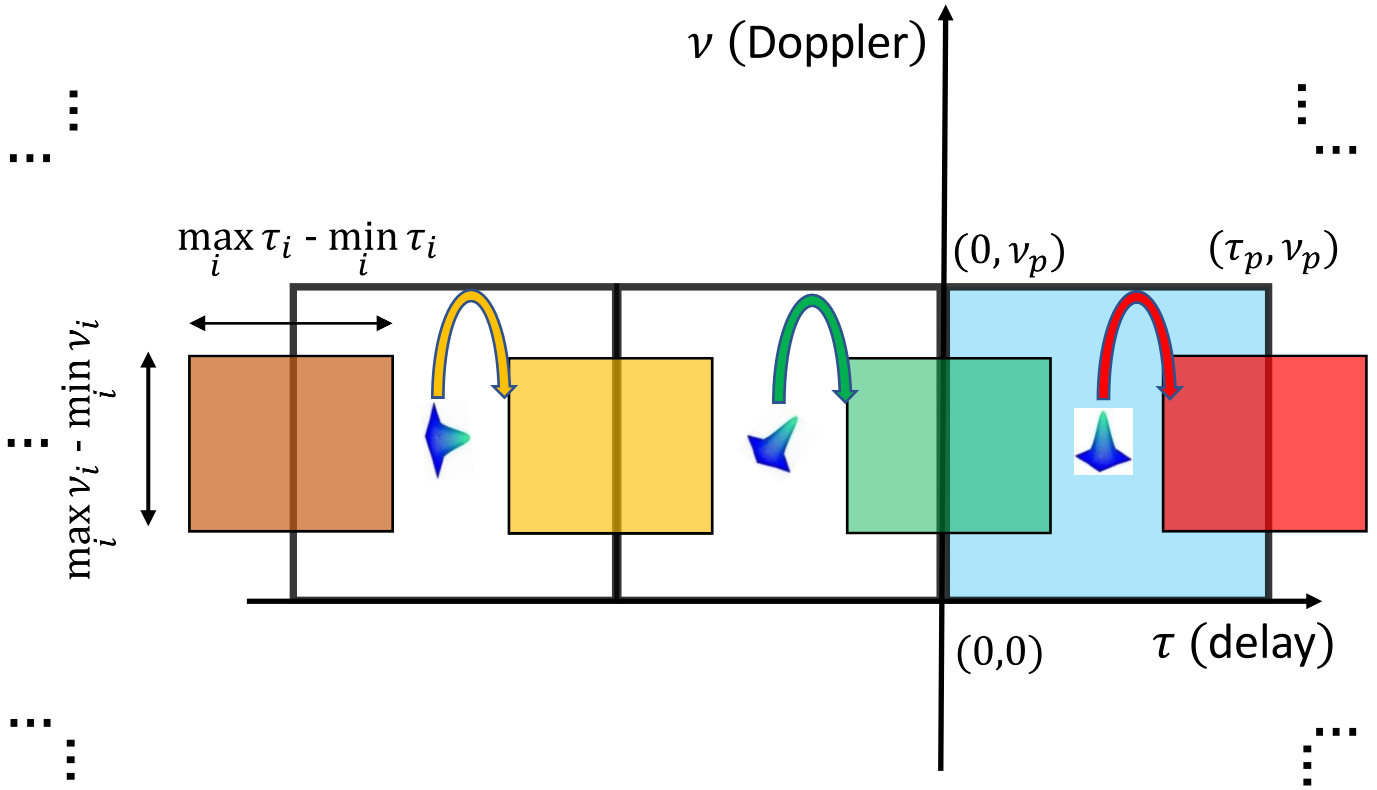


$$h(\tau,\nu)=\sum_{i=1}^{4} h_i\delta(\tau-\tau_i)\delta(\nu-\nu_i)$$

Twisted convolution: $Y(\tau,\nu)=h(\tau,\nu)*_\sigma X(\tau,\nu)$

Engineering Predictability

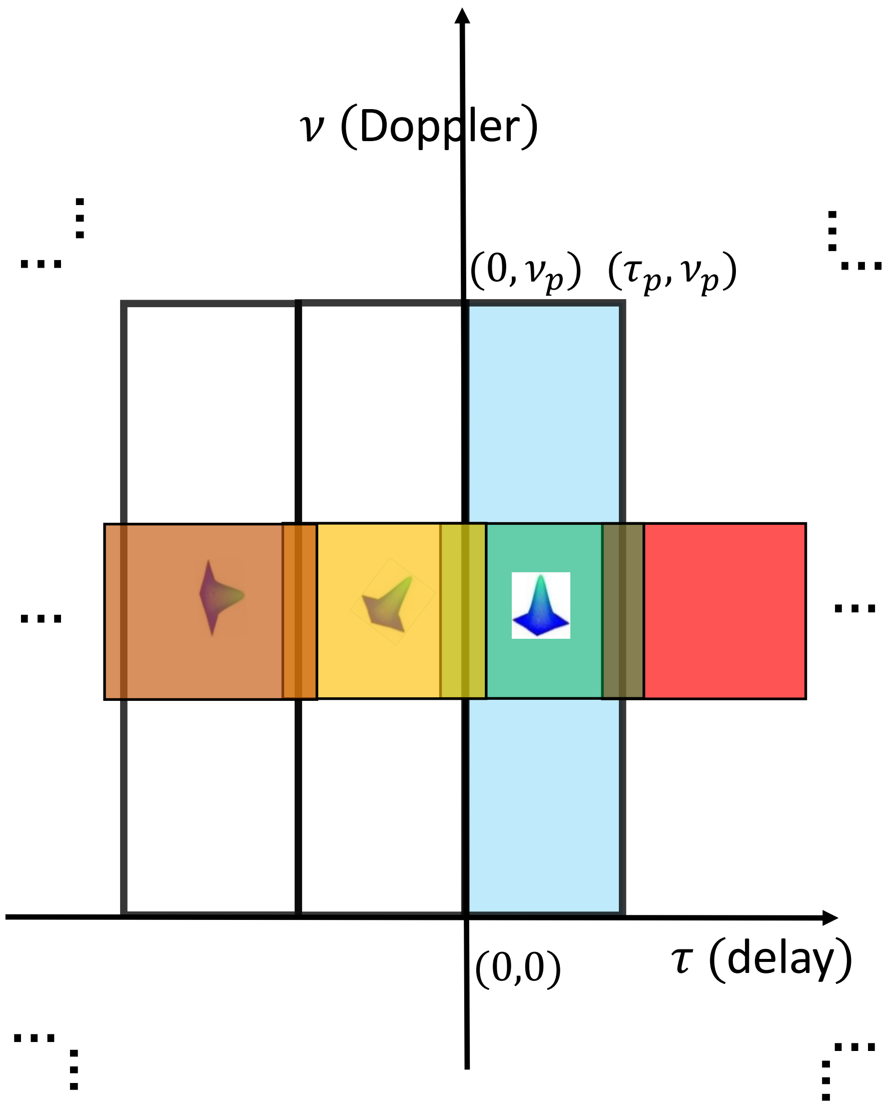
$\tau_p > \max \tau_i - \min \tau_i$ & $\nu_p > \max \nu_i - \min \nu_i$



Fundamental Period:
Blue Rectangle

No Aliasing: DD domain pulses received in the fundamental period are located within the red and the green triangles which do not overlap

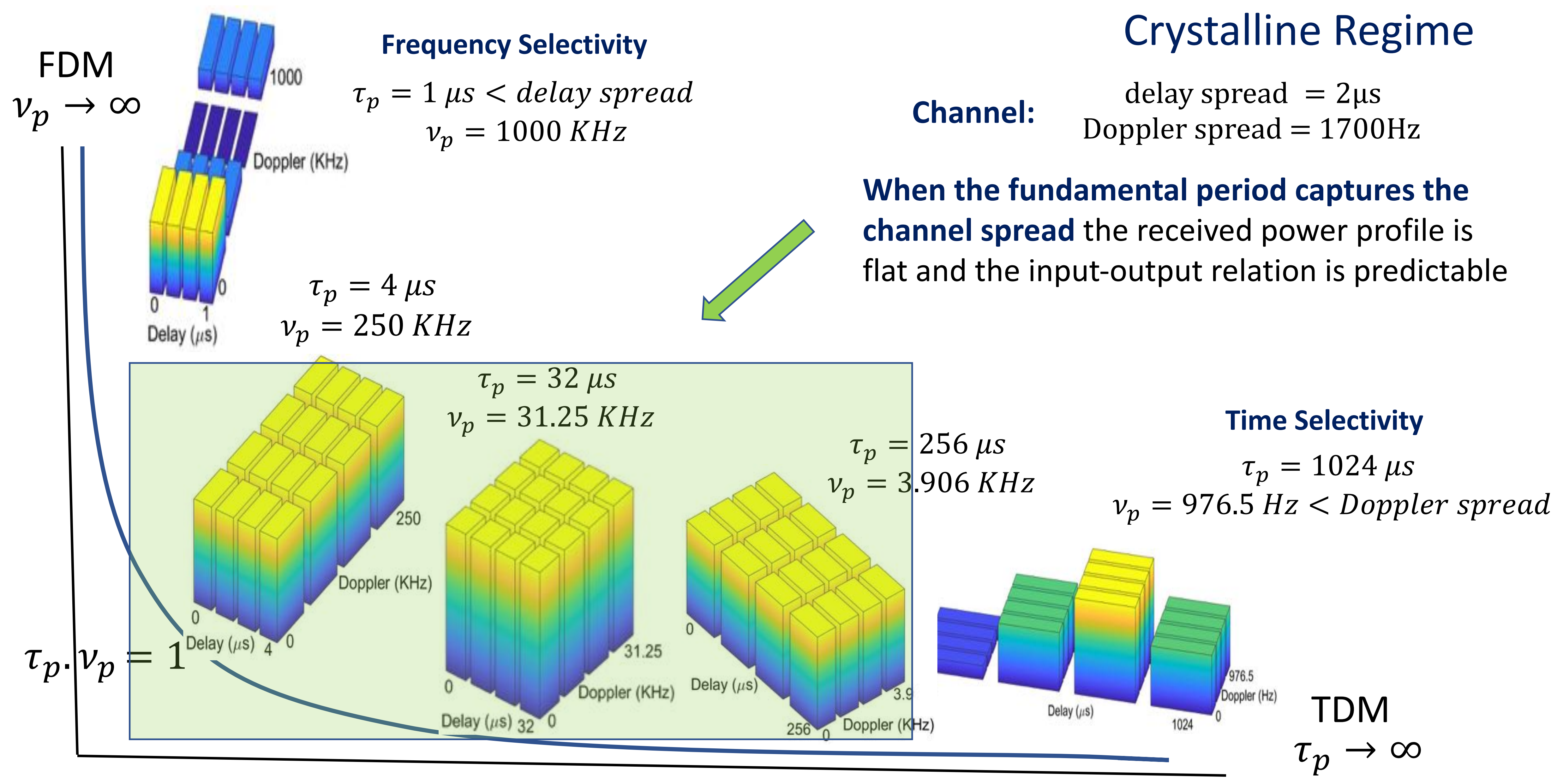
Encountering Unpredictability



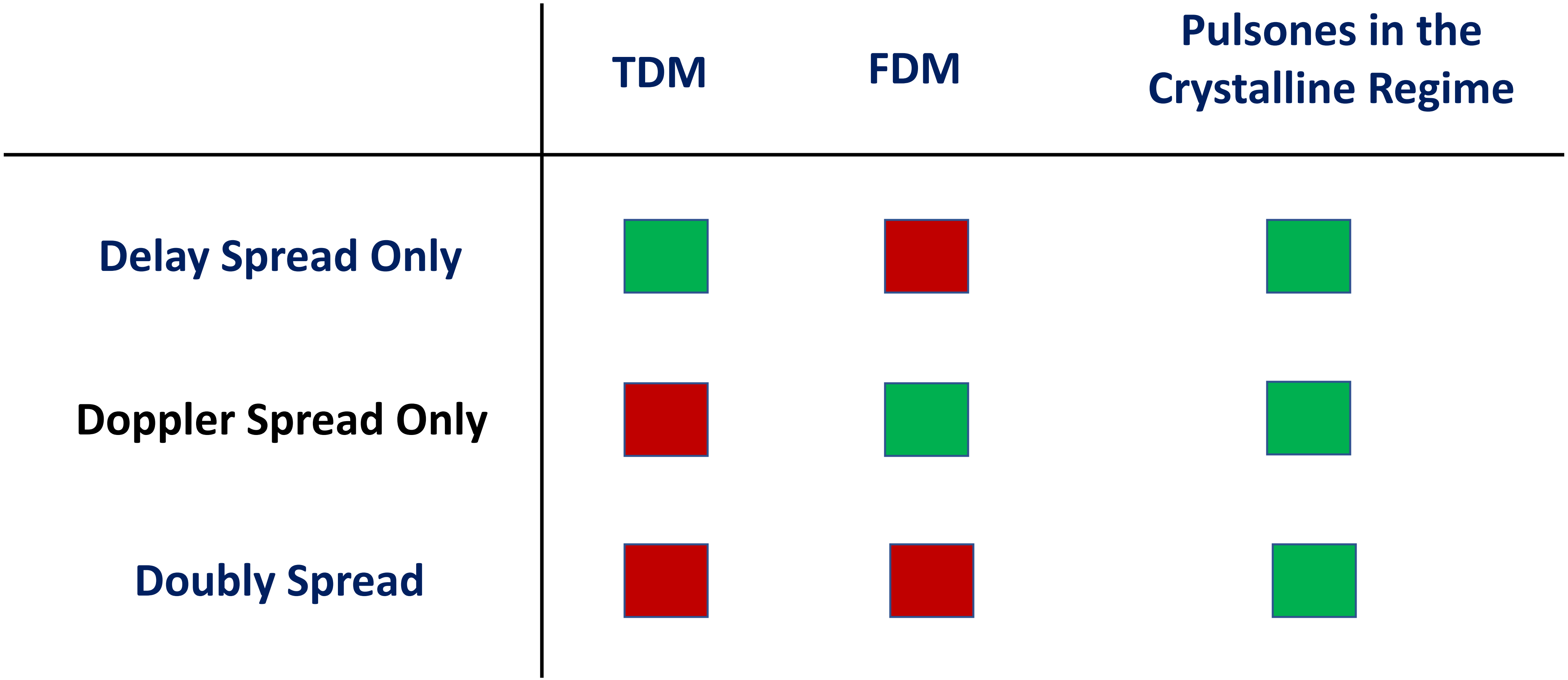
τ_p < max τ_i - min τ_i & ν_p > max ν_i - min ν_i

Fundamental Period:
Blue Rectangle

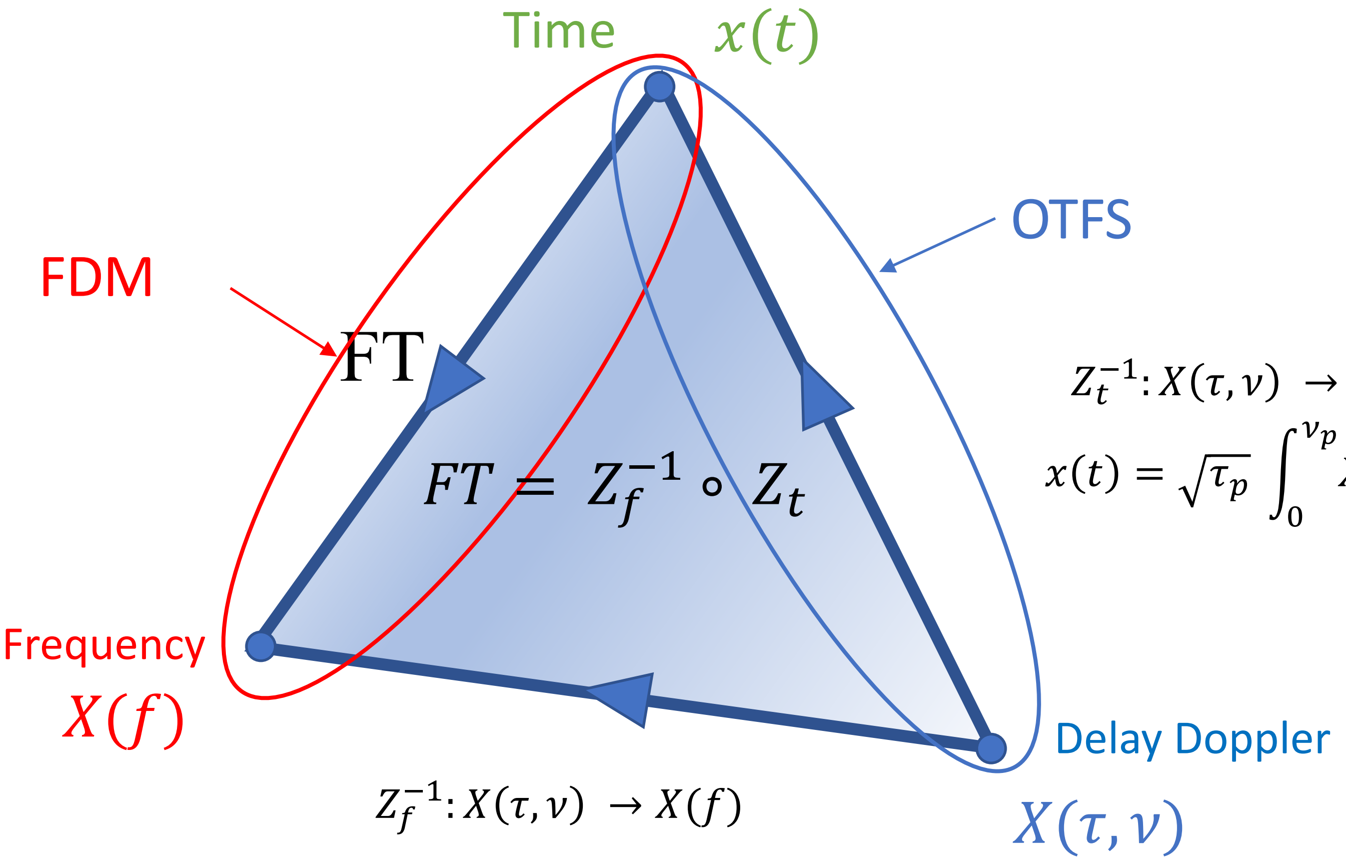
Aliasing: Small τ_p causes the green rectangle to overlap with the red and yellow rectangles, resulting in delay domain aliasing.


Crystalline Regime
Channel:
delay spread = 2μs
Doppler spread = 1700Hz
When the fundamental period captures the channel spread the received power profile is flat and the input-output relation is predictable
FDM
$\nu_p \to \infty$
Frequency Selectivity
$\tau_p = 1\ \mu s < delay\ spread$
$\nu_p = 1000\ KHz$
1000
Doppler (KHz)
0
0
1
Delay (μs)
$\tau_p = 4\ \mu s$
$\nu_p = 250\ KHz$
250
Doppler (KHz)
0
Delay (μs)
4
0
$\tau_p = 32\ \mu s$
$\nu_p = 31.25\ KHz$
31.25
Doppler (KHz)
0
Delay (μs)
32
0
$\tau_p = 256\ \mu s$
$\nu_p = 3.906\ KHz$
0
Delay (μs)
3.9
256
0
Doppler (KHz)
Time Selectivity
$\tau_p = 1024\ \mu s$
$\nu_p = 976.5\ Hz < Doppler\ spread$
976.5
Doppler (Hz)
Delay (μs)
1024
0
$\tau_p . \nu_p = 1$
TDM
$\tau_p \to \infty$

Non-Fading and Predictable Operation



Signal Processing in the DD Domain



$Z_t^{-1}: X(\tau,\nu) \rightarrow x(t)$

$$x(t) = \sqrt{\tau_p} \int_0^{\nu_p} X(t,\nu) d\nu$$

$Z_f^{-1}: X(\tau,\nu) \rightarrow X(f)$

$$\mathrm{X}(f) = \sqrt{\nu_p} \int_0^{\tau_p} X(\tau,f) e^{-j2\pi f\tau} d\tau$$

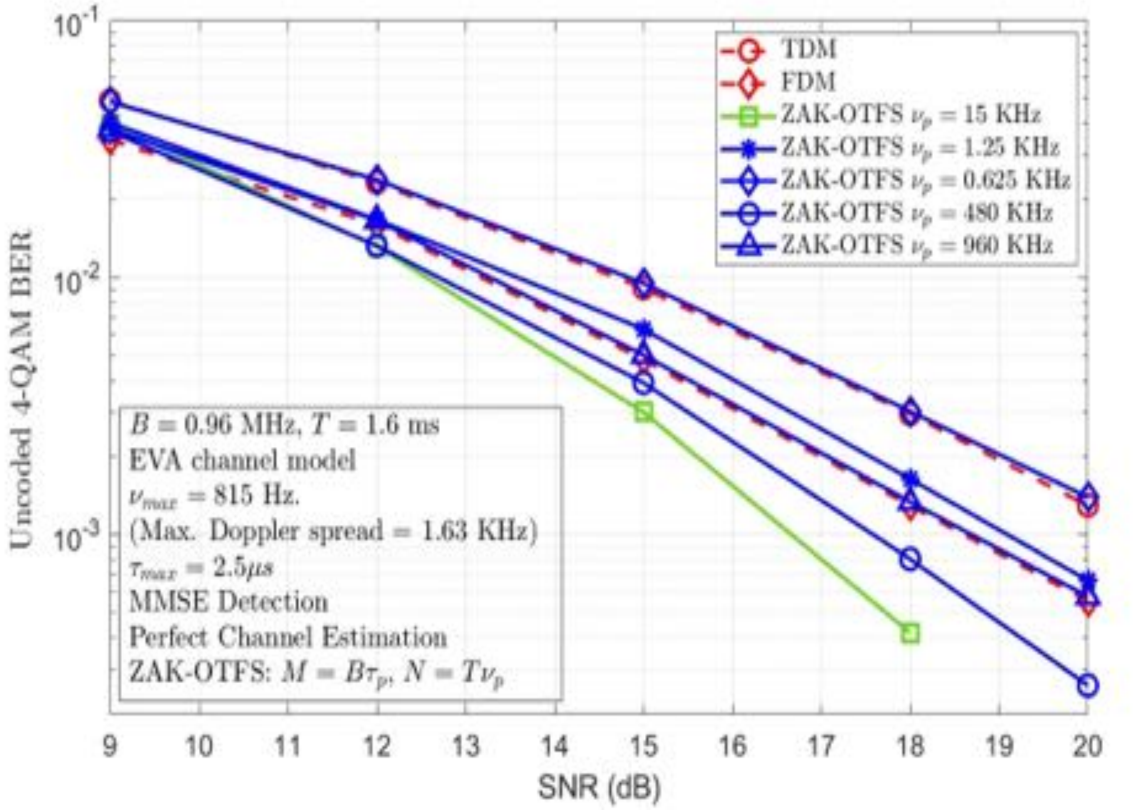
The Fourier Transform as a Composition:

First apply Z_t from TD to DD domain, then apply Z_f^{-1} from the DD domain to the FD

Not more complicated than the Fourier Transform

Impact of Fading in the Crystalline Regime

Perfect Channel Estimation: EVA Channel Model



Summary

Performance in the crystalline regime is superior

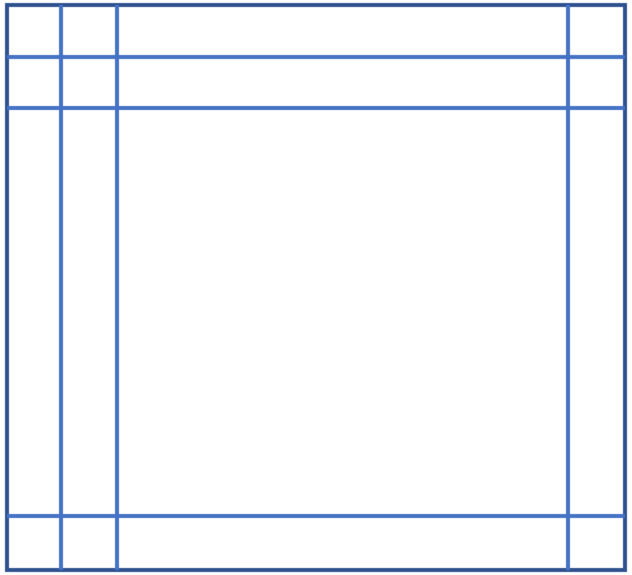
Performance approaches TDM as the delay period $\tau_p \rightarrow \infty$

Performance approaches FDM as the Doppler period $\nu_p \rightarrow \infty$

Why Model-Free ?

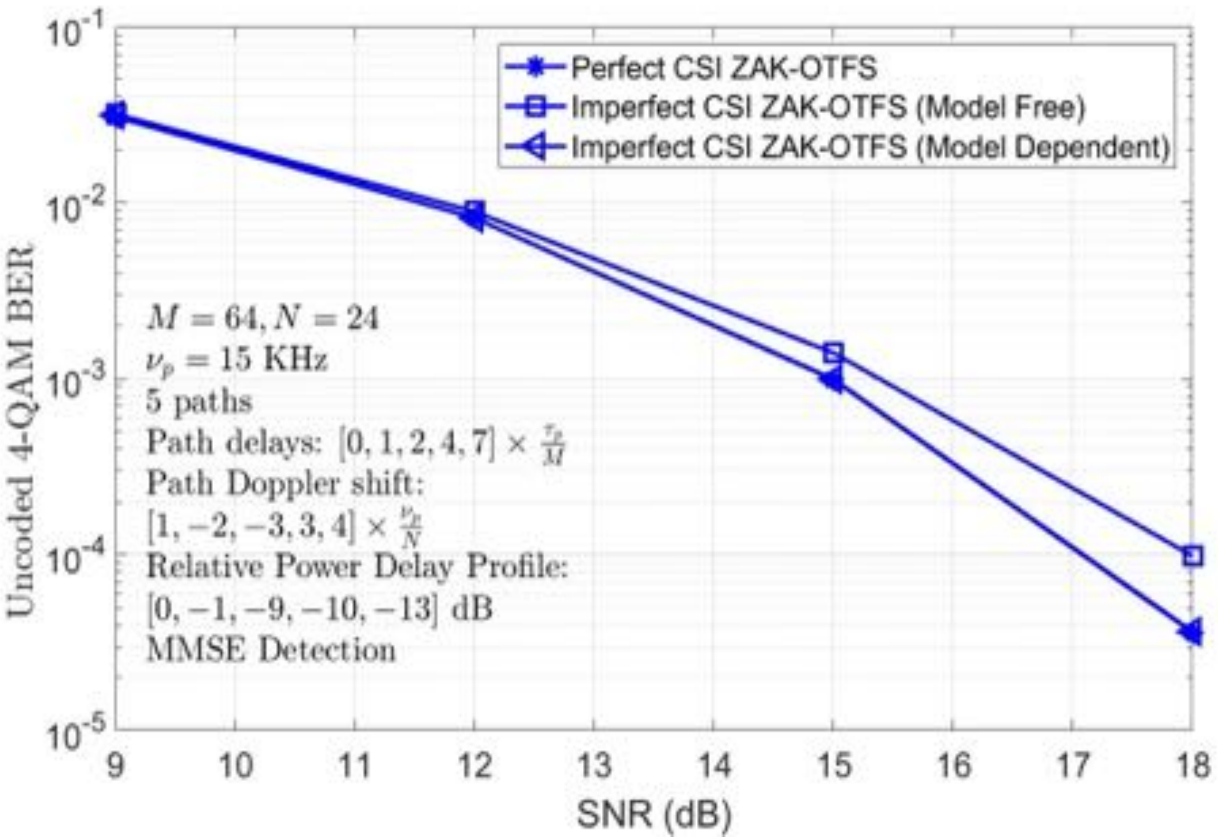
Doubly Spread Channels are becoming infinitely complicated

Input-Output Relations can be comparatively simple



Model-Free Operation: It is possible to use pulsones to learn the input-output relation directly without learning the channel

Model-Free vs Model-Dependent

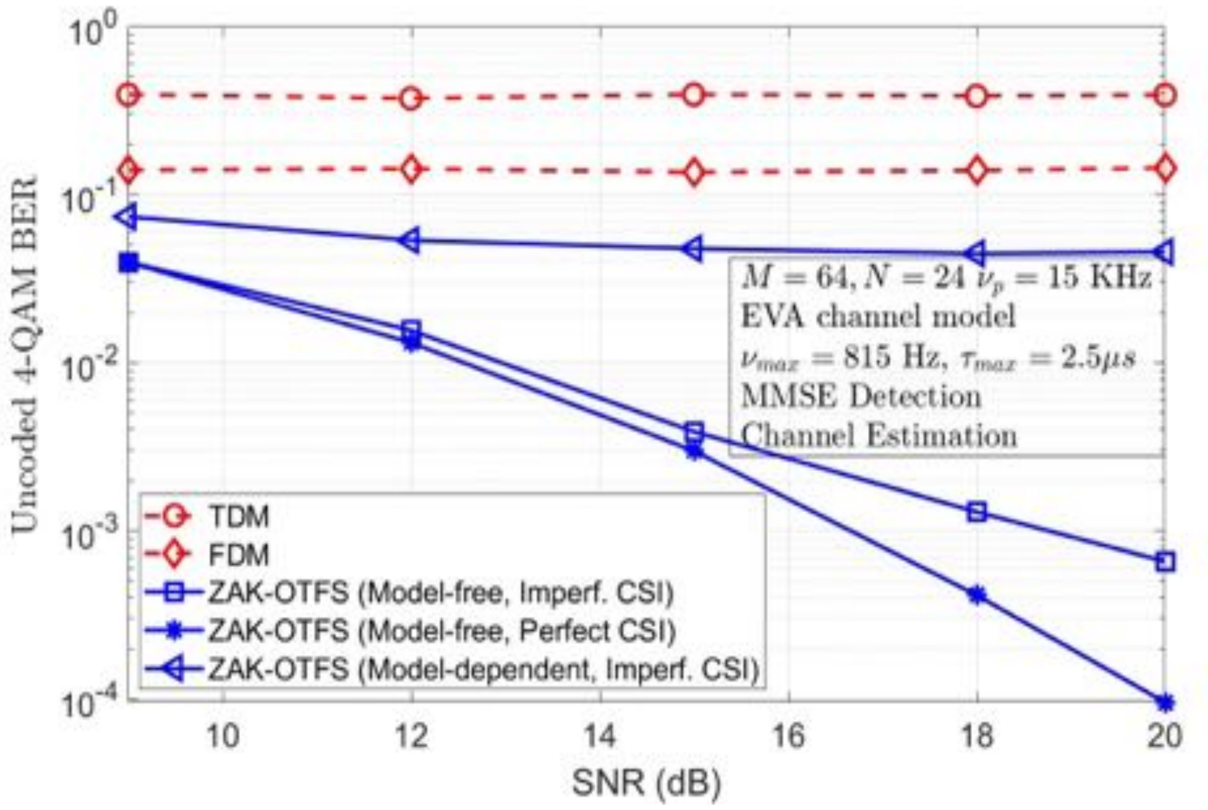


In the crystalline regime, when it is possible to learn the channel:

Model-Dependent pulsone performance coincides with ideal performance

Model-Free pulsone performance is only slightly inferior

Model-Free vs Model-Dependent

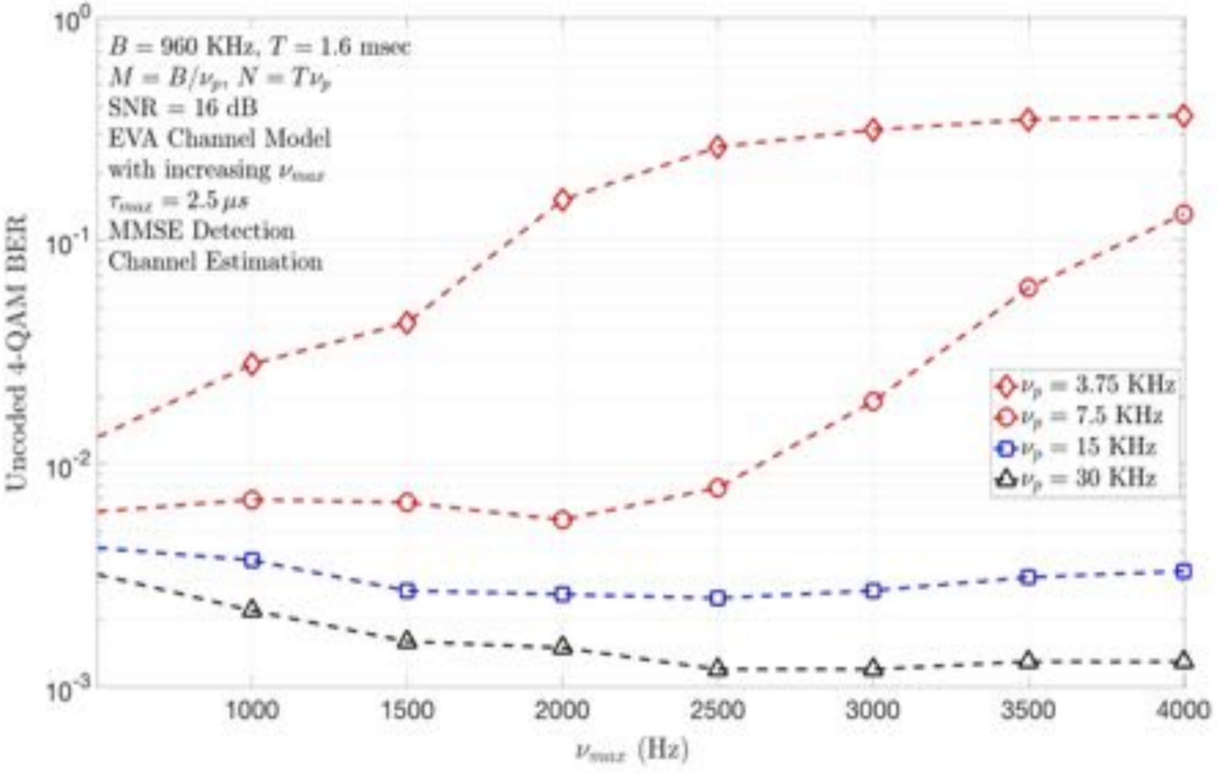


When it is not possible to learn the channel:

Pulsones support model-free operation in the crystalline regime

Not shown: Improvements in filtering – root raised cosine vs. sinc – extend the region of reliable operation

Pushing the Envelope – Impact of High Doppler



Model-Free operation in the crystalline regime:

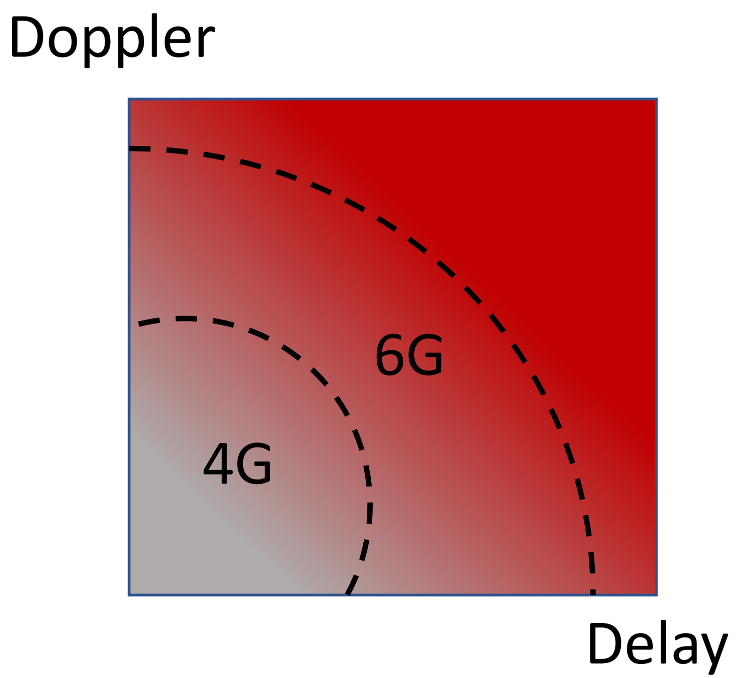
When the Doppler spread $2\nu_{max}$ is bounded away from ν_p then pulsone performance does not degrade as ν_{max} increases

When the Doppler spread $2\nu_{max}$ is close to ν_p then performance degrades because of Doppler domain aliasing

Navigating Orders of Magnitude in Doppler Spread

$$\tau_p = 32\ \mu s$$
$$\nu_p = 31.2\ KHz$$

	Delay Spread $(\boldsymbol{\mu s})$	**Doppler Spread (KHz)**
Leo-Satellite Channel	0.8	82
UAV/Aeronautical Channel (GHz)	7.0 (Take Off) 33-60 (En-Route)	7.68 (En-Route)
mmWave Mobile Channel (28GHz)	1.0	3.0
Terrestrial Mobile Channel (GHz)	5.0	0.3
Terrestrial Pedestrian Channel (GHz)	0.41	0.005



Conclusions

Bad News: It is becoming impossible to learn channels

Good News: It is still very possible to learn input-output relations

Pulsones enable model-free operation in the crystalline regime, opening the door to machine learning

What makes this possible? We are using the operators that define doubly spread channels both to probe the channel, and to transmit information

Live here, You must

Pulsones in the Crystalline Regime