Model-Aided Data Driven Adaptive Target Detection for Channel Matrix-Based Cognitive Radar

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Signals, Information, Inference, & Learning (SIIL) Group

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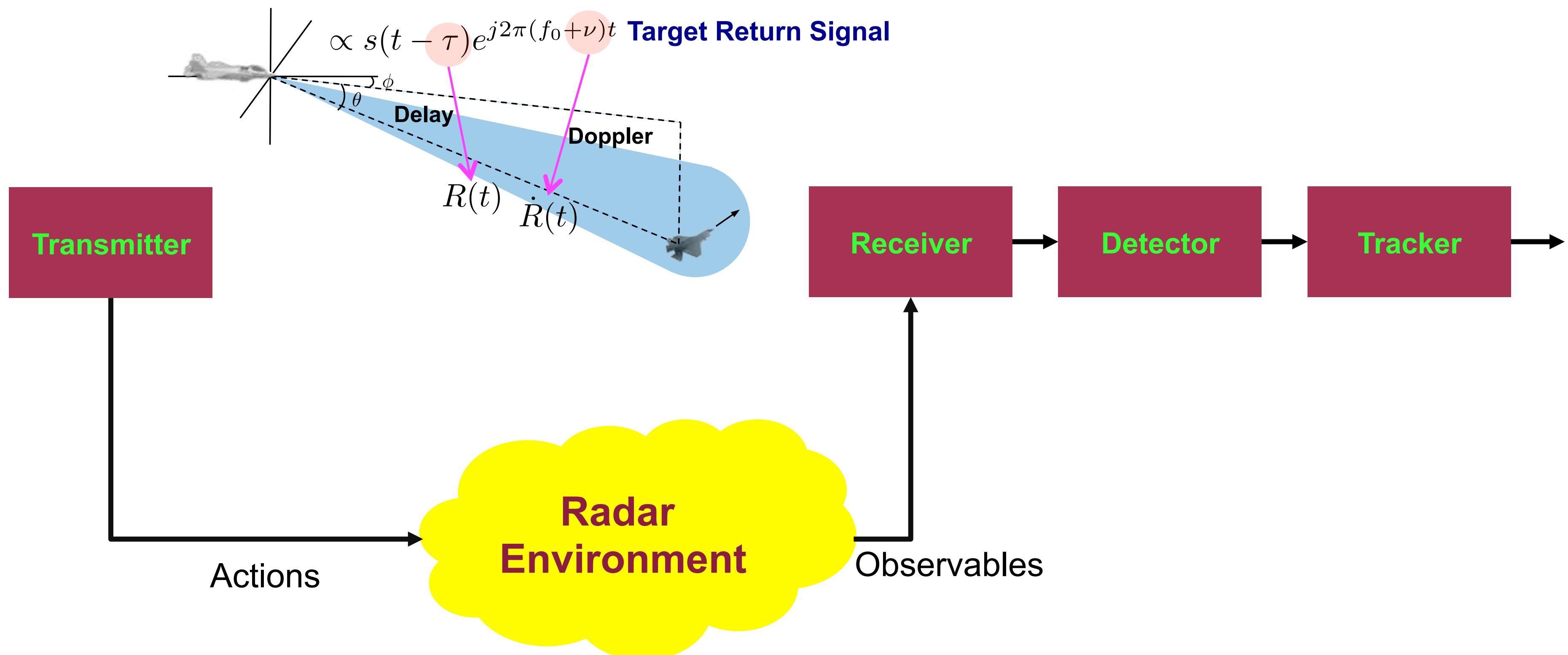
Outline

- **Adaptive Target Detection for Conventional Radar**
- Adaptive Target Detection for Cognitive Radar
- Deep Neural Network (DNN) based Target Detection
- Summary

Conventional Radar: A Feed-Forward System



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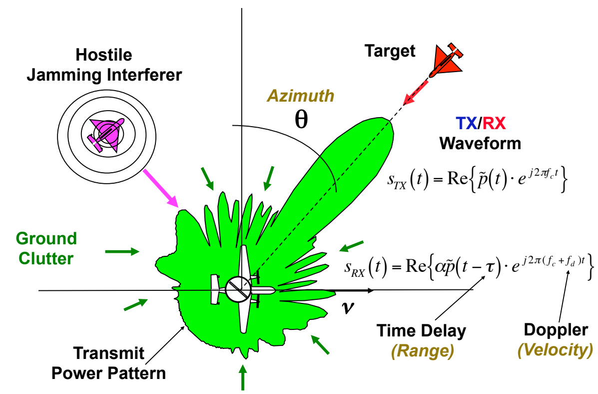
Conventional Radar Data Model

- **Radar with array of l elements transmitting waveform** $s(t)$
- **Radar data often modeled as complex Gaussian:** $\mathbf{x} \sim \mathcal{CN}(\alpha \cdot \mathbf{v}, \mathbf{R})$
- $E\{\mathbf{x}\} = \alpha \cdot \mathbf{v}$ ➡ **Signal (Target)**
  - $\alpha = \alpha\,[\mathbf{s}(t)]$ **Target complex amplitude**
  - $\mathbf{v}$ **Target space-time response:** $\mathbf{v}_{RADAR} = \mathbf{v}(\varphi, \theta, f_d)$,

    Azimuth, Elevation (φ, θ); **Doppler** (f_d)
- $\mathrm{cov}(\mathbf{x}) = \mathbf{R}$ ➡ **Noise + Interference**
  - $\mathbf{R} = \mathbf{R}_{System} + \mathbf{R}_{Jammer} + \mathbf{R}_{Clutter}$
  - $\mathbf{R}_{Clutter} = \mathbf{R}_{Clutter}[\mathbf{s}(t)]$ ➡ **Depends on Tx Waveform**
  - **Thus, waveform dependence is nonlinear**



P. Setlur and M. Rangaswamy, "Waveform design for radar stap in signal dependent interference," IEEE Transactions on Signal Processing, vol. 64, no. 1, pp. 19-34, 2015.

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Conventional Model:
Adaptive Array Detection Problem

- **Radar with array of l elements**
- **Test data vector $\mathbf{x}$**
- **Look for presence of target in $\mathbf{x}$:**

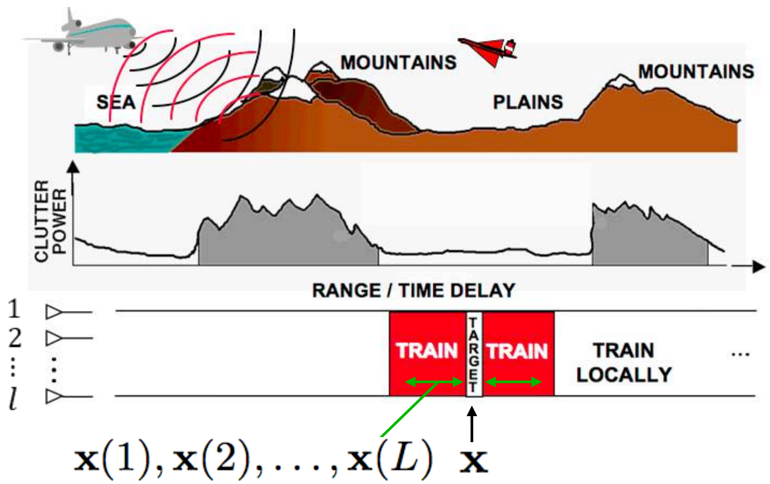
$$H_0 : \mathbf{x} = \mathbf{n}, \qquad \mathbf{x} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$$
$$H_1 : \mathbf{x} = \alpha \cdot \mathbf{v} + \mathbf{n}, \quad \mathbf{x} \sim \mathcal{CN}(\alpha \cdot \mathbf{v}, \mathbf{R})$$

- **Unknowns are α and $\mathbf{R}$ with possible error in $\mathbf{v}$**
- **L Training noise only data vectors**

$$\mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(L) \quad L \geq l$$
$$\mathbf{x}(k) \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$$

- **Desire viable detection statistic:** $t(\mathbf{x}, \mathbf{x}(1), \mathbf{x}(2), \ldots, \mathbf{x}(L), \mathbf{v})$

Example: Radar Environment



Reed, Irving S., John D. Mallett, and Lawrence E. Brennan. "Rapid convergence rate in adaptive arrays." *IEEE Transactions on Aerospace and Electronic Systems* 6 (1974): 853-863.

Conventional Model: Summary of Adaptive Detection Algorithms

- **Adaptive Matched Filter (AMF)**
  Robey, et. al. IEEE T-AES 1992
  Reed & Chen 1992, Reed et. al. 1974

$$t_{AMF} = \frac{|\mathbf{v}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}|^2}{\mathbf{v}^H \widehat{\mathbf{R}}^{-1} \mathbf{v}}$$

- **Generalized Likelihood Ratio Test (GLRT)**
  Kelly IEEE T-AES 1986, Khatri & Rao 1985

$$t_{GLRT} = \frac{t_{AMF}}{1 + \mathbf{x}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}}$$

- **Adaptive Coherence Estimator (ACE)**
  Conte et. al. IEEE T-AES 1995,
  Scharf Asil. 1996, Kraut IEEE T-SP 2001

$$t_{ACE} = \frac{t_{AMF}}{\mathbf{x}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}}$$

- **Adaptive Sidelobe Blanker (ASB)**
  Kreithen, Baranoski, 1996
  Richmond Asilomar 1997& 1998,
  Richmond IEEE T-SP 2000

$$f(t_{AMF}, t_{ACE})$$

Each Algorithm is Function of Sample Covariance

$$\widehat{\mathbf{R}} = \mathbf{x}(1)\mathbf{x}^H(1) + \cdots + \mathbf{x}(L)\mathbf{x}^H(L)$$

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- **Adaptive Target Detection for Cognitive Radar**
- Deep Neural Network (DNN) based Target Detection
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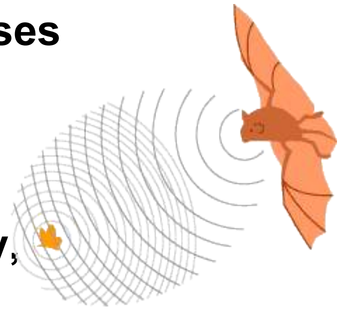
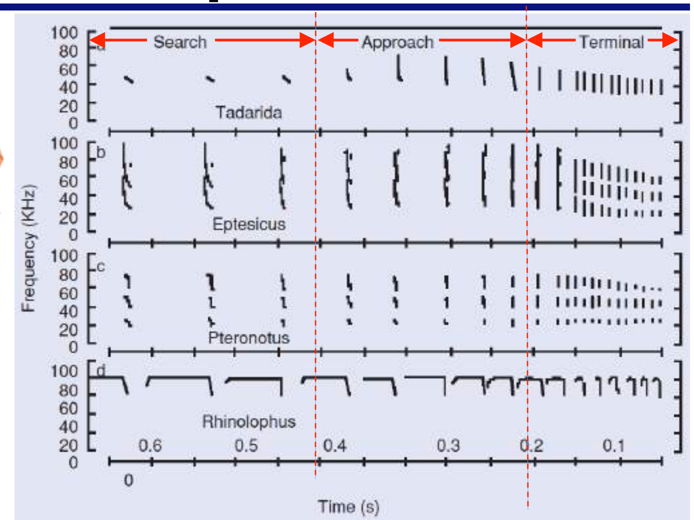
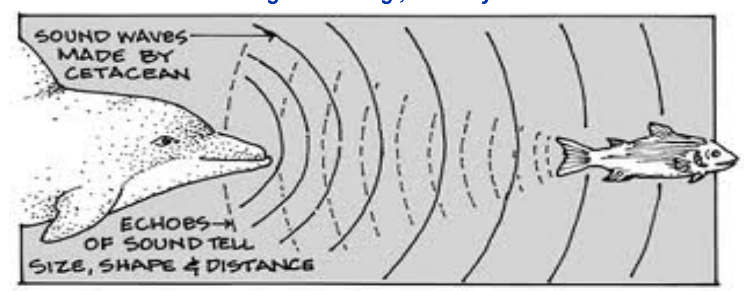
Inspiration for Cognitive Systems: Echo Location / Human Vision-Perception

- **Bats** and **dolphins** emit short sound pulses to locate food
- Return echoes inform about type of prey, range and bearing
- **Waveform** characteristics **adapted** as bat / dolphin closes on prey
  - Higher pulse repetitions
  - Changes in chirp rates
- Human **visual brain**
  - Cerebral cortex major player in cognition (Fuster model introduced)

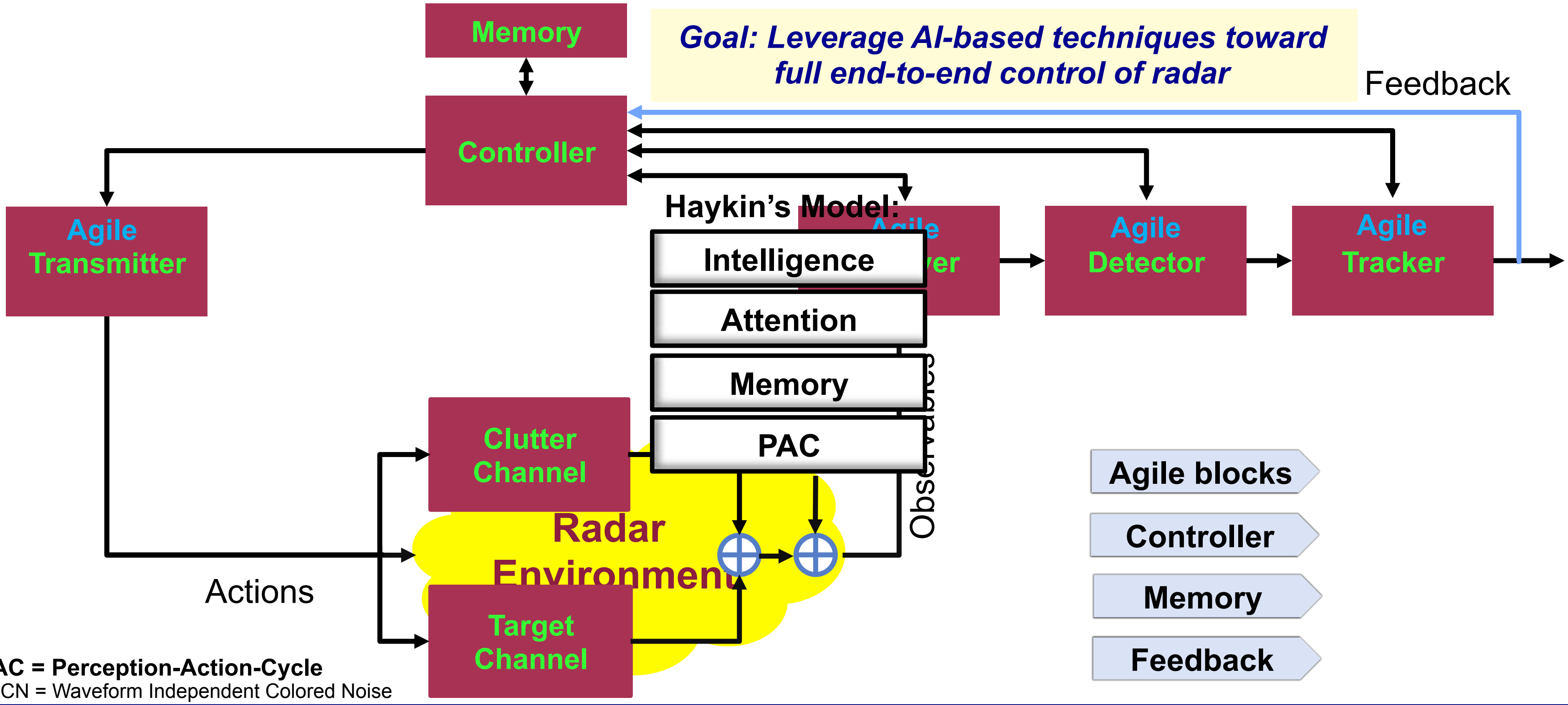




S. Haykin, "Cognitive radar: a way of the future," IEEE Sig. Proc. Mag., January 2006.



Cognitive Radar: Adaptation via a Feedback System



PAC = Perception-Action-Cycle
WICN = Waveform Independent Colored Noise

S. Haykin, "Cognitive radar: a way of the future," IEEE signal processing magazine, vol. 23, no. 1, pp. 30{40, 2006.
J. Guerci, "Cognitive radar: The knowledge-aided fully adaptive approach." *Artech House radar library. Artech House*, 2010.

Motivating Channel Matrix-Based Radar Model

- **LTV system output:** $r(t) = \int \tilde{h}(t,a)s(a)da = -\int \tilde{h}(t,t-\tau)s(t-\tau)d\tau \triangleq \int h(t,\tau)s(t-\tau)d\tau$
  $a = t-\tau,\ da = -d\tau,$ $\quad h(t,\tau) \triangleq -\tilde{h}(t,t-\tau)$
- **Consider Fourier representation** $h(t,\tau) = \int H(\tau,\nu)e^{j2\pi\nu t}d\nu \Longrightarrow r(t) = \int\int H(\tau,\nu)s(t-\tau)e^{j2\pi\nu t}d\nu d\tau$
- **Thus,** $r(t) \simeq \sum_i \sum_m H_{i,m}s(t-\tau_i)e^{j2\pi\nu_m t}$. **Discretizing yields form*** $r[n] = \sum_i \sum_m \mathcal{H}[i,m]s[n-i]e^{j\frac{2\pi m}{M}n}$
- **Note that** $r[n] = \sum_k \sum_m \mathcal{H}[n-k,m]s[k]e^{j\frac{2\pi m}{M}n} = \sum_k \left(\sum_m \mathcal{H}[n-k,m]e^{j\frac{2\pi m}{M}n}\right)\cdot s[k] \triangleq \sum_k \tilde{\mathcal{H}}_n[k]s[k]$
  $k = n-i,\ i = n-k$
- **Each received data sample is linear transformation of waveform:**

$$\begin{bmatrix} r[0] \\ r[1] \\ \vdots \\ r[N-1] \end{bmatrix} = \begin{bmatrix} \star & \cdots & \star \\ \vdots & \ddots & \vdots \\ \star & \cdots & \star \end{bmatrix} \begin{bmatrix} s[0] \\ s[1] \\ \vdots \\ s[N-1] \end{bmatrix} + \begin{bmatrix} \tilde{n}[0] \\ \tilde{n}[1] \\ \vdots \\ \tilde{n}[N-1] \end{bmatrix} \Rightarrow \mathbf{r} = \mathbf{H}\mathbf{s} + \mathbf{n}$$

Channel Matrix — **Waveform** — **Noise + Interference**

- **Linear model can capture dominant effects of doubly spread multipath channel**

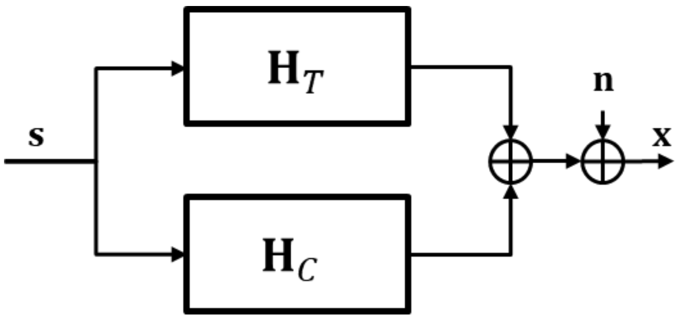
LTV = linear time-varying

*B. Friedland, Trans. Amer. Instit. Elec. Eng., Part II: Applic. & Industry, vol. 75, no. 6, pp. 407-414, Jan. 1957

Channel Matrix-based
Data Model Used in Cognitive Radar*

- **Target produces scattering component, say $\mathbf{H}_T\mathbf{s}$**
- **Clutter / reverberation also yields scattering component**
  - **Can model in similar way, i.e. as $\mathbf{H}_C\mathbf{s}$**
- **Thus, we have the general MIMO radar data model:**

$$\mathbf{x} = \mathbf{H}_T\mathbf{s} + \mathbf{H}_C\mathbf{s} + \mathbf{n}$$



  - **Same TX waveforms produce both target return and clutter**
  - **Scattering propagation paths are different, however, i.e. $\mathbf{H}_T \neq \mathbf{H}_C$**
- **Simplifies waveform optimization problem**
- **Although model is linear, fundamental research questions remained unaddressed**

*J. R. Guerci, Cognitive Radar: The Knowledge-Aided Fully Adaptive Approach, Artech House, Inc., Norwood, MA, 2010.

Cognitive Radar Target Detection Problem

- **Binary hypothesis test:**

No Target $\Longrightarrow H_0 : \mathbf{x} = \mathbf{H}_C\mathbf{s} + \mathbf{n} \quad , \mathbf{x} \sim \mathcal{CN}(\mathbf{H}_C\mathbf{s}, \mathbf{R})$

Target Present $\Longrightarrow H_1 : \mathbf{x} = \mathbf{H}_C\mathbf{s} + \mathbf{H}_T\mathbf{s} + \mathbf{n} \quad , \mathbf{x} \sim \mathcal{CN}(\mathbf{H}_C\mathbf{s} + \mathbf{H}_T\mathbf{s}, \mathbf{R})$

- **Waveform independent colored noise (WICN)** $\mathbf{n} \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$
- $\mathbf{H}_T, \mathbf{H}_C$ **and** $\mathbf{R}$ **known**
  - **Likelihood Ratio Test (LRT)**
- $\mathbf{H}_T, \mathbf{H}_C$ **and** $\mathbf{R}$ **unknown** ➡ **Composite Hypothesis testing**
  - **Average Likelihood Ratio Test (ALRT)***
    - **Channel matrices modeled as random**
  - **Generalized Likelihood Ratio Test (GLRT)****
    - **Channel matrices assumed deterministic but unknown during the observation time**

GLRT and ALRT establish benchmarks for comparison with machine / deep learning-based cognitive radar detection, and guides AI architectures

*T. Ali and C. D. Richmond, “Optimal target detection for random channel matrix-based cognitive radar/sonar,” in 2021 IEEE Radar Conference (RadarConf21), 2021, pp. 1–6.
**T. Ali, A. S. Bondre, C. D. Richmond, "Adaptive Detection Algorithms for Channel Matrix-Based Cognitive Radar/Sonar," 2022 IEEE Radar Conference (RadarConf22), 2022, pp. 1-6.

Cognitive Radar Data Model: Adaptive Detection Problem Formulation

- **Motivated by GLRT approach taken by Kelly***
- K **secondary** and M **primary data vectors,** $L_s = K + M$.
- **Total received data matrix** $\mathbf{X} \in \mathbb{C}^{l \times L_s}$

$$\mathbf{X} = [\underbrace{\mathbf{x}(1)|\cdots|\mathbf{x}(K)}_{\text{Secondary Data}}|\underbrace{\mathbf{x}(K+1)|\cdots|\mathbf{x}(L_s)}_{\text{Primary Data}}] = [\mathbf{X}_S|\mathbf{X}_P] \qquad \mathbf{X}_S \in \mathbb{C}^{l \times K} \quad \mathbf{X}_P \in \mathbb{C}^{l \times M}$$

Secondary Data **(Clutter + noise only)**

Primary Data **(May contain target as well)**

- **Secondary data** ➡ $\mathbf{x}(k) = \mathbf{H}_C\mathbf{s}(k) + \mathbf{n}(k), k = 1, \ldots, K$
- **Primary data** ➡

$$\left.\begin{array}{l} H_0 : \mathbf{x}(k) = \mathbf{H}_C\mathbf{s}(k) + \mathbf{n}(k) \\ H_1 : \mathbf{x}(k) = \mathbf{H}_C\mathbf{s}(k) + \mathbf{H}_T\mathbf{s}(k) + \mathbf{n}(k) \end{array}\right\} k = K+1, \cdots, L_s$$

- **Waveform matrix** $\mathbf{S} = [\mathbf{s}(1)|\cdots|\mathbf{s}(K)|\mathbf{s}(K+1)|\cdots|\mathbf{s}(L_s)] = [\mathbf{S}_S \quad \mathbf{S}_P] \in \mathbb{C}^{l \times L_s}$
- **WICN** $\mathbf{n}(k)$ **I.I.D with** $\mathbf{n}(k) \sim \mathcal{CN}(\mathbf{0}, \mathbf{R})$, $k = 1, \cdots, L_s$.

*E. J. Kelly, "An adaptive detection algorithm," IEEE Transactions on Aerospace and Electronic systems, no. 2, pp. 115–127, 1986.

GLRT for Cognitive Radar Data Model: Unknown $\mathbf{R}, \mathbf{H}_T$ & $\mathbf{H}_C$

- **Channel matrices** $\mathrm{H}_C, \mathrm{H}_T$ **and WICN covariance** R **are all unknown**
- **The GLRT is given by:** $\Lambda_{GLRT}(\mathbf{X}) = \dfrac{\max_{\mathbf{R},\mathbf{H}_C,\mathbf{H}_T} f_1(\mathbf{X};\mathbf{R},\mathbf{H}_C,\mathbf{H}_T)}{\max_{\mathbf{R},\mathbf{H}_C} f_0(\mathbf{X};\mathbf{R},\mathbf{H}_C)}$
- **PDF under** H_0

$$f_0(\mathbf{X};\mathbf{H}_C,\mathbf{R}) = \left[\frac{1}{\pi^l|\mathbf{R}|}\right]^{L_s} \exp\left\{-\sum_{k=1}^{L_s}\left(\mathbf{x}(k)-\mathbf{H}_C\mathbf{s}(k)\right)^H\mathbf{R}^{-1}\left(\mathbf{x}(k)-\mathbf{H}_C\mathbf{s}(k)\right)\right\}.$$

- **PDF under** H_1

$$f_1(\mathbf{X};\mathbf{H}_C,\mathbf{H}_T,\mathbf{R}) = \left[\frac{1}{\pi^l|\mathbf{R}|}\right]^{L_s} \exp\left\{-\sum_{k=1}^{K}\left(\mathbf{x}(k)-\mathbf{H}_C\mathbf{s}(k)\right)^H\mathbf{R}^{-1}\left(\mathbf{x}(k)-\mathbf{H}_C\mathbf{s}(k)\right) - \sum_{k=K+1}^{L_s}\left(\mathbf{x}(k)-(\mathbf{H}_C+\mathbf{H}_T)\mathbf{s}(k)\right)^H\mathbf{R}^{-1}\left(\mathbf{x}(k)-(\mathbf{H}_C+\mathbf{H}_T)\mathbf{s}(k)\right)\right\}$$

GLRT for Cognitive Radar Data Model: Unknown $\mathbf{R}, \mathbf{H}_T$ & $\mathbf{H}_C$*

$$\tilde{\Lambda}_{GLRT}(\mathbf{X}) = \frac{|\mathbf{X}\mathfrak{P}(\mathbf{S}^H|\mathbf{I})\mathbf{X}^H|}{|\mathbf{X}_P\mathfrak{P}(\mathbf{S}_P^H|\mathbf{I})\mathbf{X}_P^H + \mathbf{X}_S\mathfrak{P}(\mathbf{S}_S^H|\mathbf{I})\mathbf{X}_S^H|} \underset{H_0}{\overset{H_1}{\gtrless}} \tilde{\eta}_{GLRT}$$

- **With noiseless input data** $\mathbf{X}_P = (\mathbf{H}_C + \mathbf{H}_T)\mathbf{S}_P$, $\mathbf{X}_S = \mathbf{H}_C\mathbf{S}_S \Longrightarrow$
  - $\mathbf{X}_S\mathfrak{P}(\mathbf{S}_S^H|\mathbf{I})\mathbf{X}_S^H$ **nullifies everything in clutter subspace in sec. data**
  - $\mathbf{X}_P\mathfrak{P}(\mathbf{S}_P^H|\mathbf{I})\mathbf{X}_P^H$ **nullifies everything in clutter+target subspaces in pri. data**

  Denominator approximates $|\mathbf{R}|$

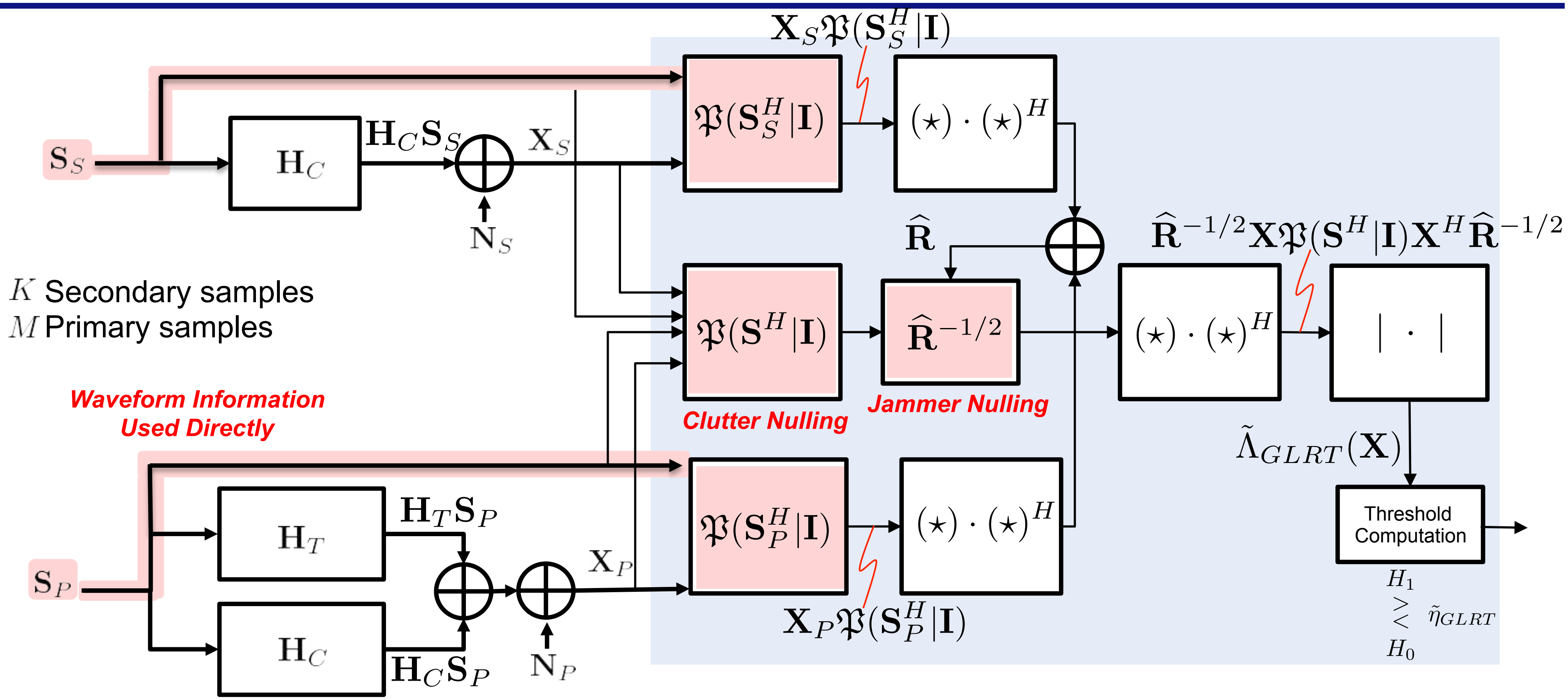
  - $\mathbf{X}\mathfrak{P}(\mathbf{S}^H|\mathbf{I})\mathbf{X}^H$ **cancels everything in clutter subspaces in both pri. and sec. data sets and in target subspace in pri. data set except the residue:** $\mathbf{H}_T\mathbf{S}_P[\mathbf{I} - \mathbf{S}_P^H(\mathbf{S}\mathbf{S}^H)^{-1}\mathbf{S}_P]\mathbf{S}_P^H\mathbf{H}_T^H$
- **Maximum-likelihood estimates of clutter channel matrices:**

  $\widehat{\mathbf{H}}_{C0} = \mathbf{X}\mathbf{S}^H(\mathbf{S}\mathbf{S}^H)^{-1}$ **(under H0)** $\widehat{\mathbf{H}}_{C1} = \mathbf{X}_S\mathbf{S}_S^H(\mathbf{S}_S\mathbf{S}_S^H)^{-1}$ **(under H1)**
- **GLRT statistic depends only on measured data and waveform (desire to optimize)**

$\mathfrak{P}(\mathbf{G}|\mathbf{I}) = \mathbf{I} - \mathbf{G}(\mathbf{G}^H\mathbf{G})^{-1}\mathbf{G}^H$

*T. Ali, A. S. Bondre, C. D. Richmond, IEEE Radar Conference, New York, March 21—25, 2022.

GLRT Architecture for Cognitive Radar Detection



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- Adaptive Target Detection for Cognitive Radar
- **Deep Neural Network (DNN) based Target Detection**
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Model Aided DNN Design

Model-based Methods

- Aware of the statistical model
- Requires smaller amounts of data
- Sensitive to model inaccuracies
- Algorithms balance complexity and optimality

Model-Aided DNN Methods

- Hybrid approach that employs data driven techniques aided by knowledge from model-based approaches
- Reduce required training
- Improve convergence rates.

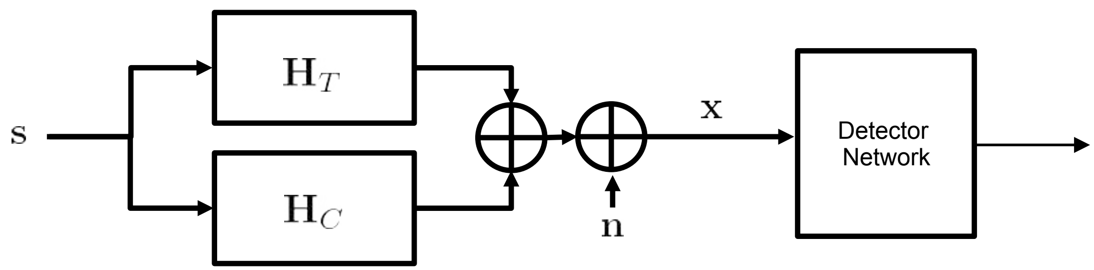
DNN-based Methods

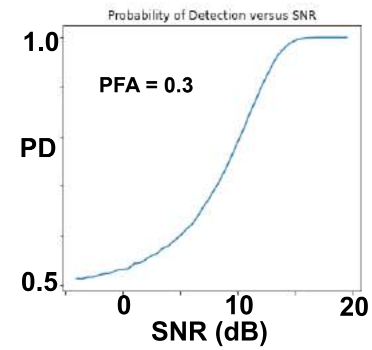
- Model-free and data driven
- Difficulty generalizing and may overfit
- Requires large amounts of training set
- Sensitive to training

B Tolooshams , N Shlezinger, D Ba, Y Eldar ,"Tutorial on Model Based Deep Learning in Signal Processing", ICASSP 2021

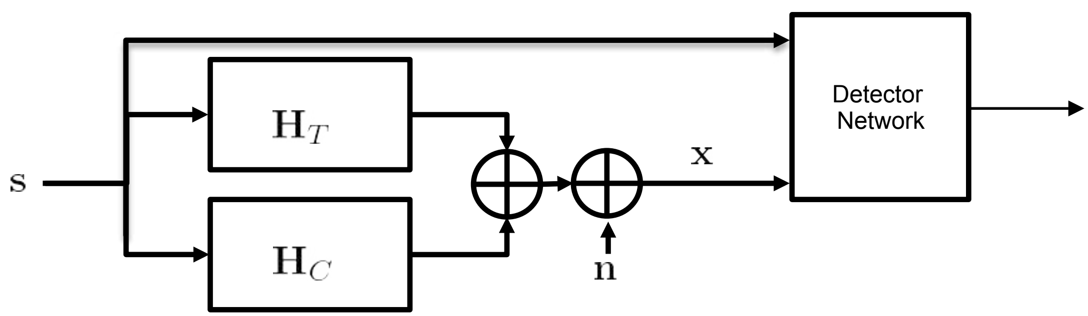
DNN Based Detector: First Attempts

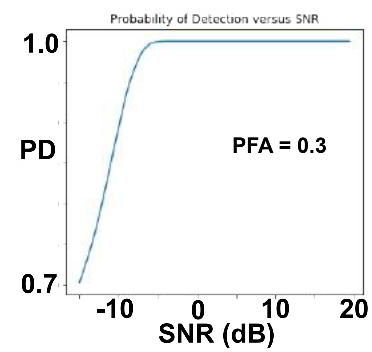
- **We tried training DNN to classify data as “target bearing” or “target free”:**





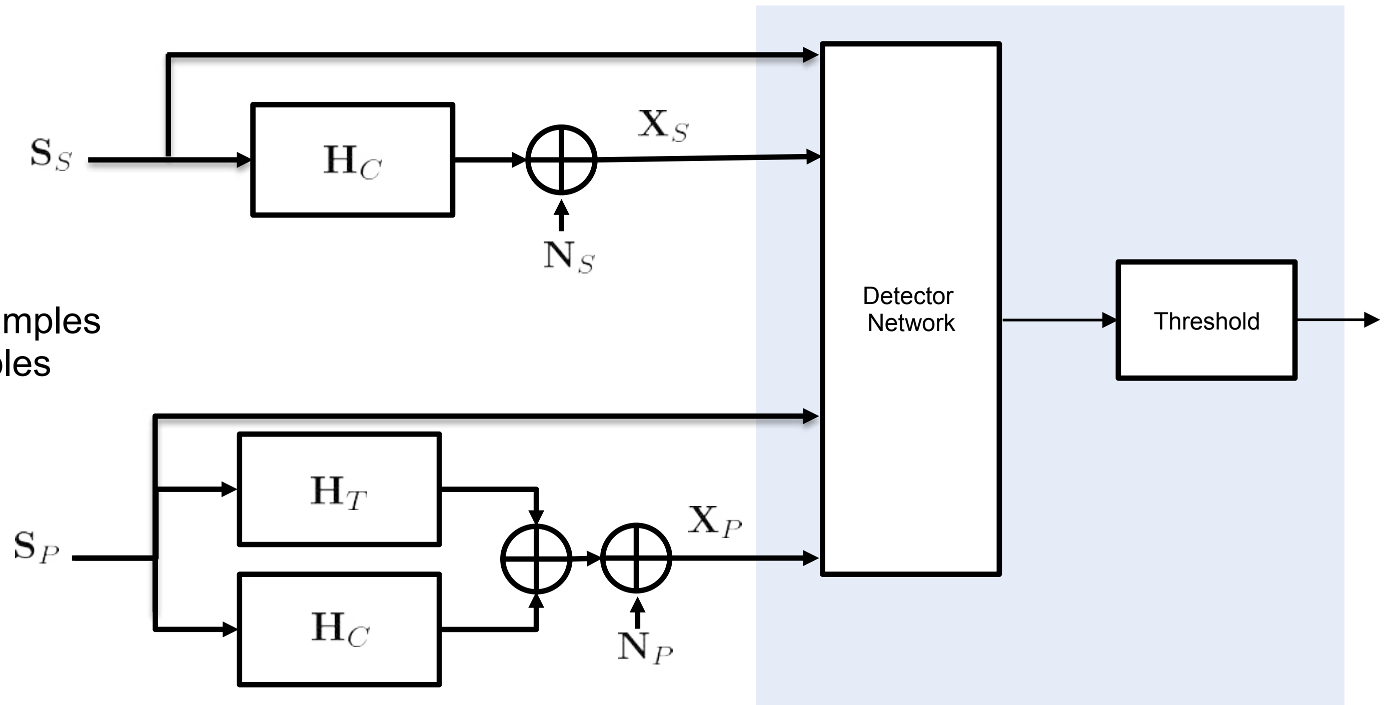
- **GLRT exploits knowledge of waveform. Thus, we tried:**





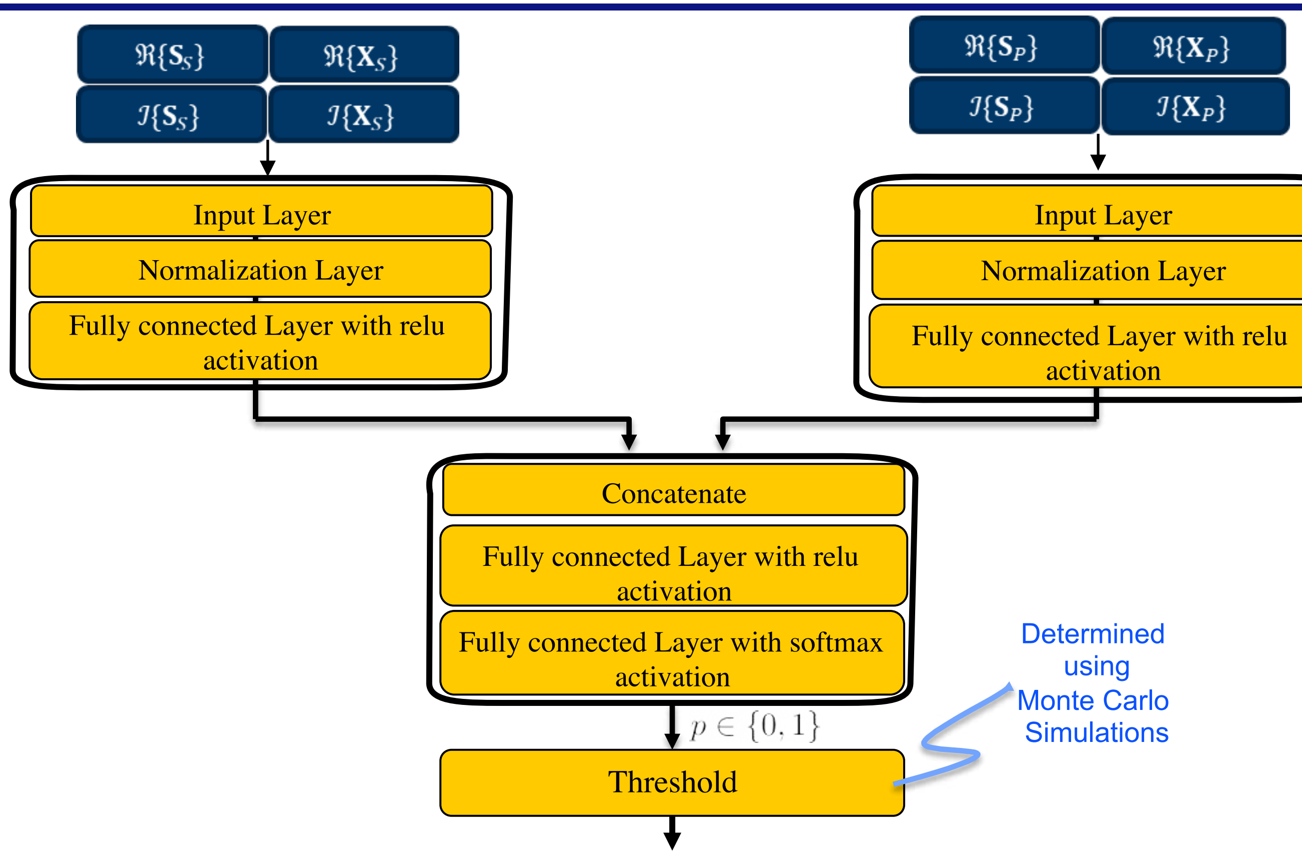
DNN Based Adaptive Detector: Architecture

K Secondary samples
M Primary samples



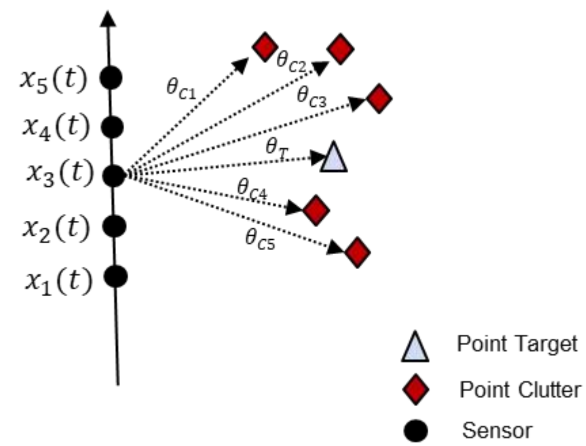
- **Waveform history provided as input to the DNN**

DNN Based Adaptive Detector: Network Layers



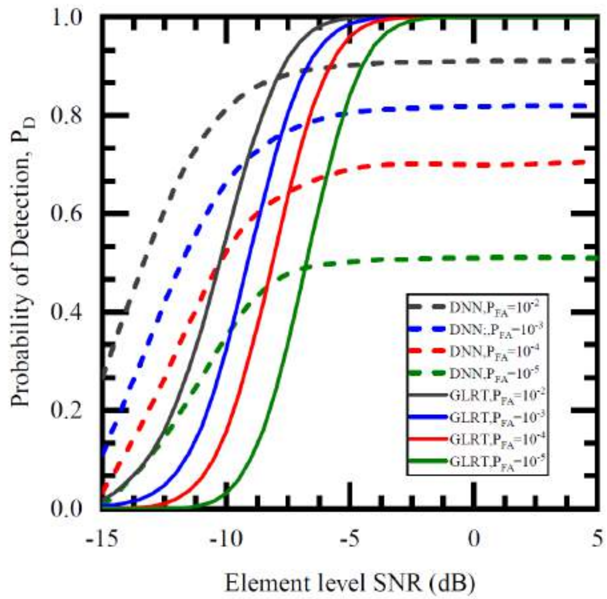
Simulation Results

- **ULA of $l = 5$ sensors**
- **Single Point Target with zero doppler and five discrete point clutter placed randomly relative to the boresight of antenna array i.e.** $\theta_T, \theta_{Ci} \sim \mathcal{U}(-\pi/4, \pi/4),\ i = 1, 2, \cdots, 5$
- **Zero mean Complex Gaussian Noise**
- **Rician Model of the Channel Matrices**
- **Waveform selected from Complex Gaussian distribution**

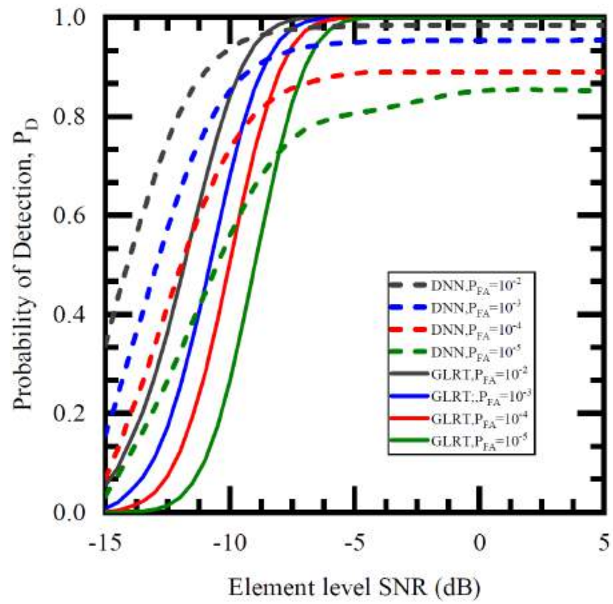


Simulation Results

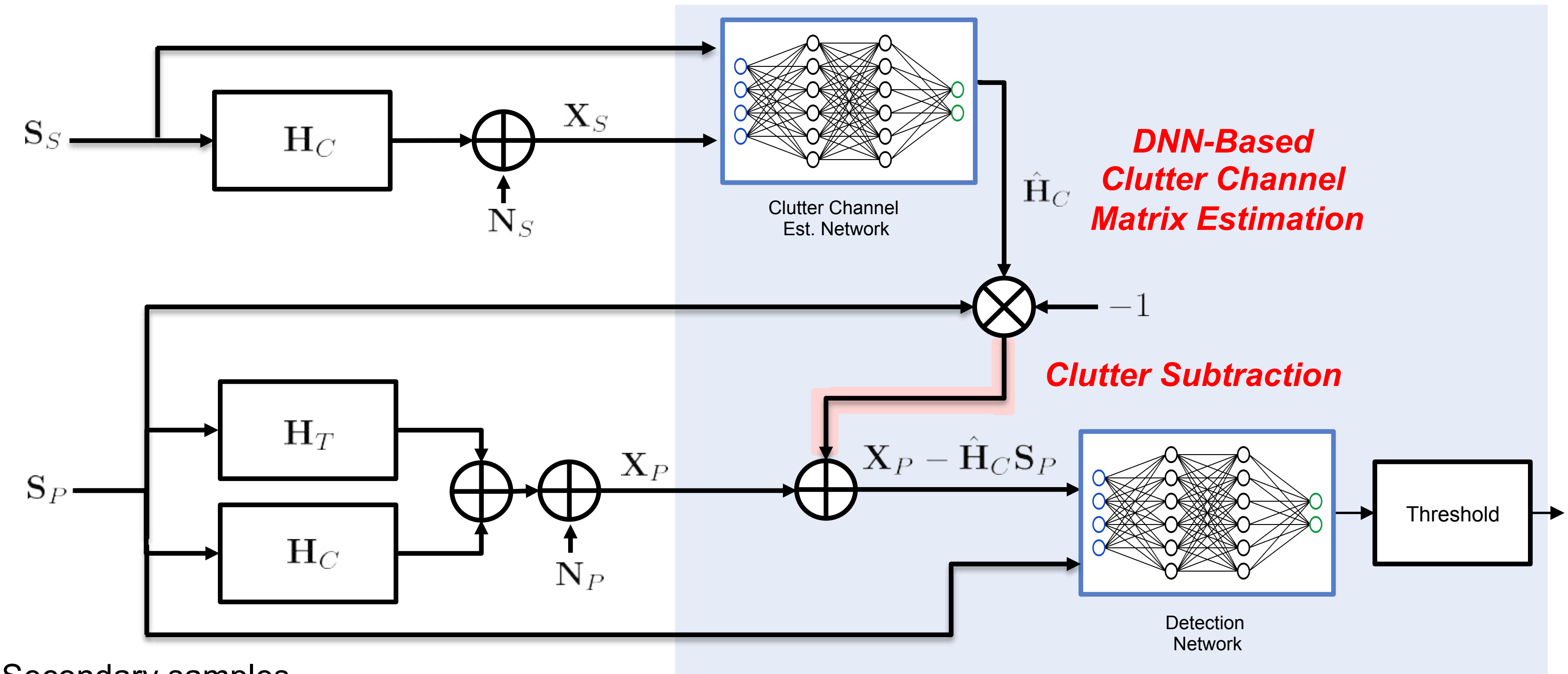
DNN Detector vs GLRT, K=25, M=5



DNN Detector vs GLRT, K=25, M=10

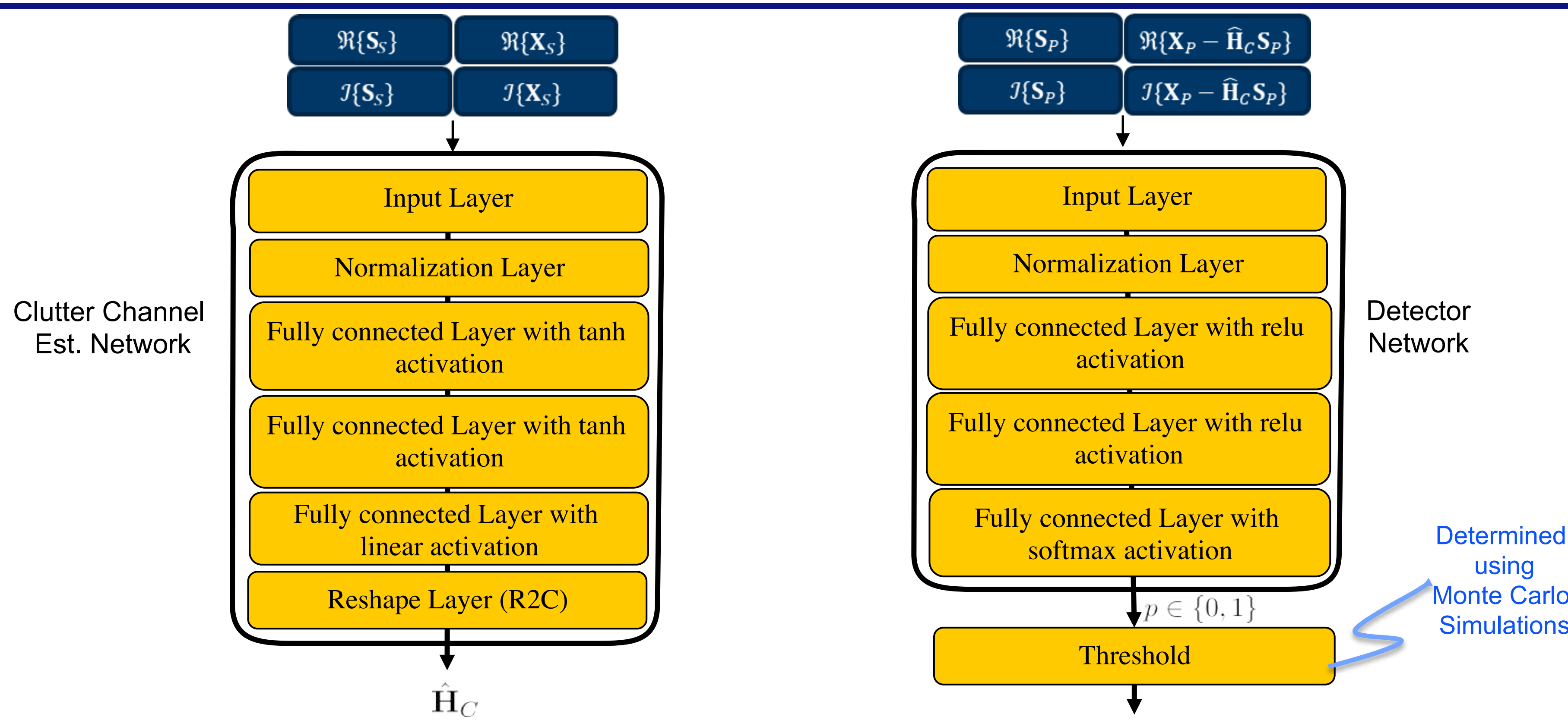


Model Aided DNN Detector: Clutter Removal



K Secondary samples
M Primary samples

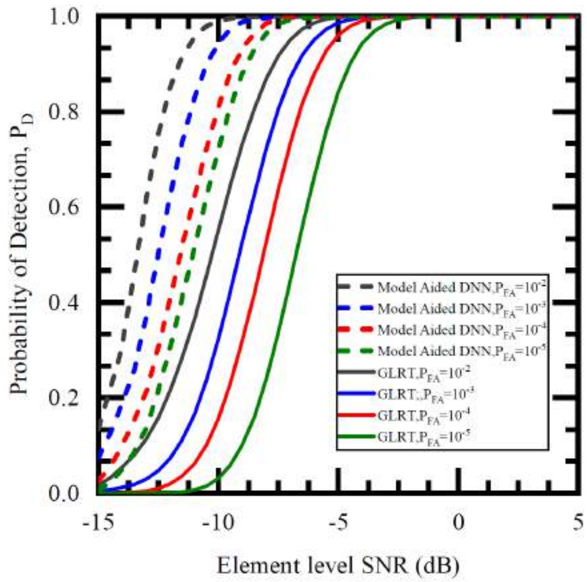
Model Aided DNN based Detector



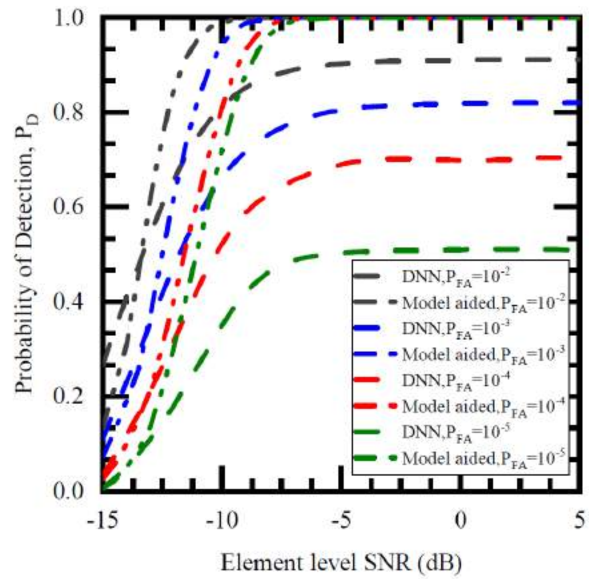
DNN Network Details

Simulation Results

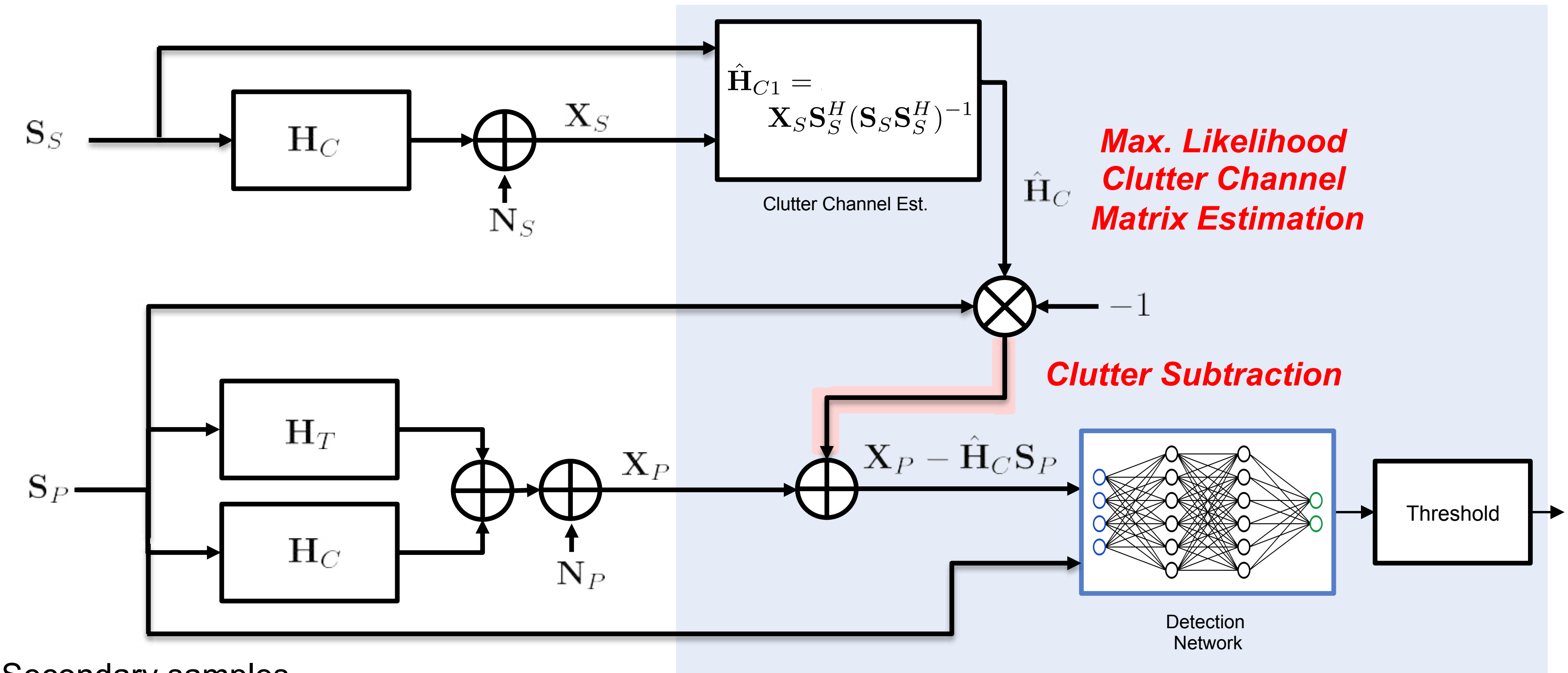
Model aided DNN Detector vs GLRT, K=25, M=5



Model aided vs model free DNN Detector, K=25, M=05



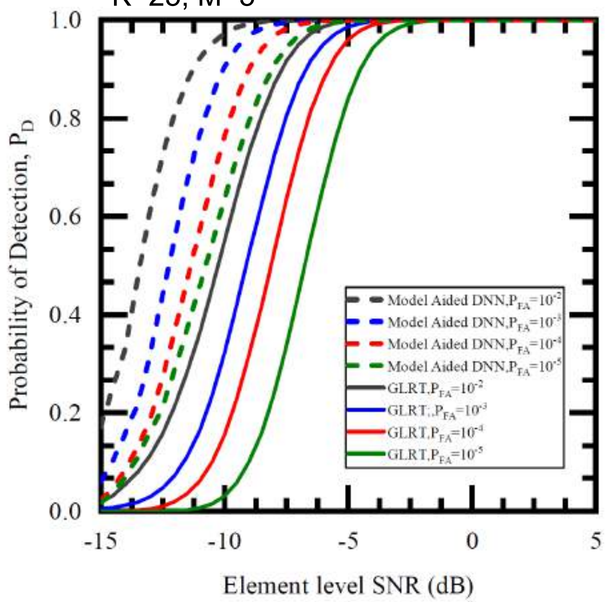
Model Aided DNN based Detector



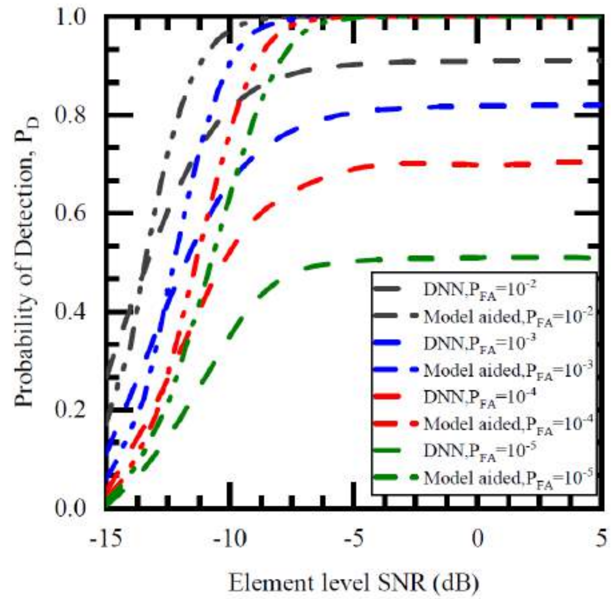
K Secondary samples
M Primary samples

Simulation Results

Model aided DNN Detector vs GLRT, K=25, M=5



Model aided vs Model free DNN Detector, K=25, M=5



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Summary

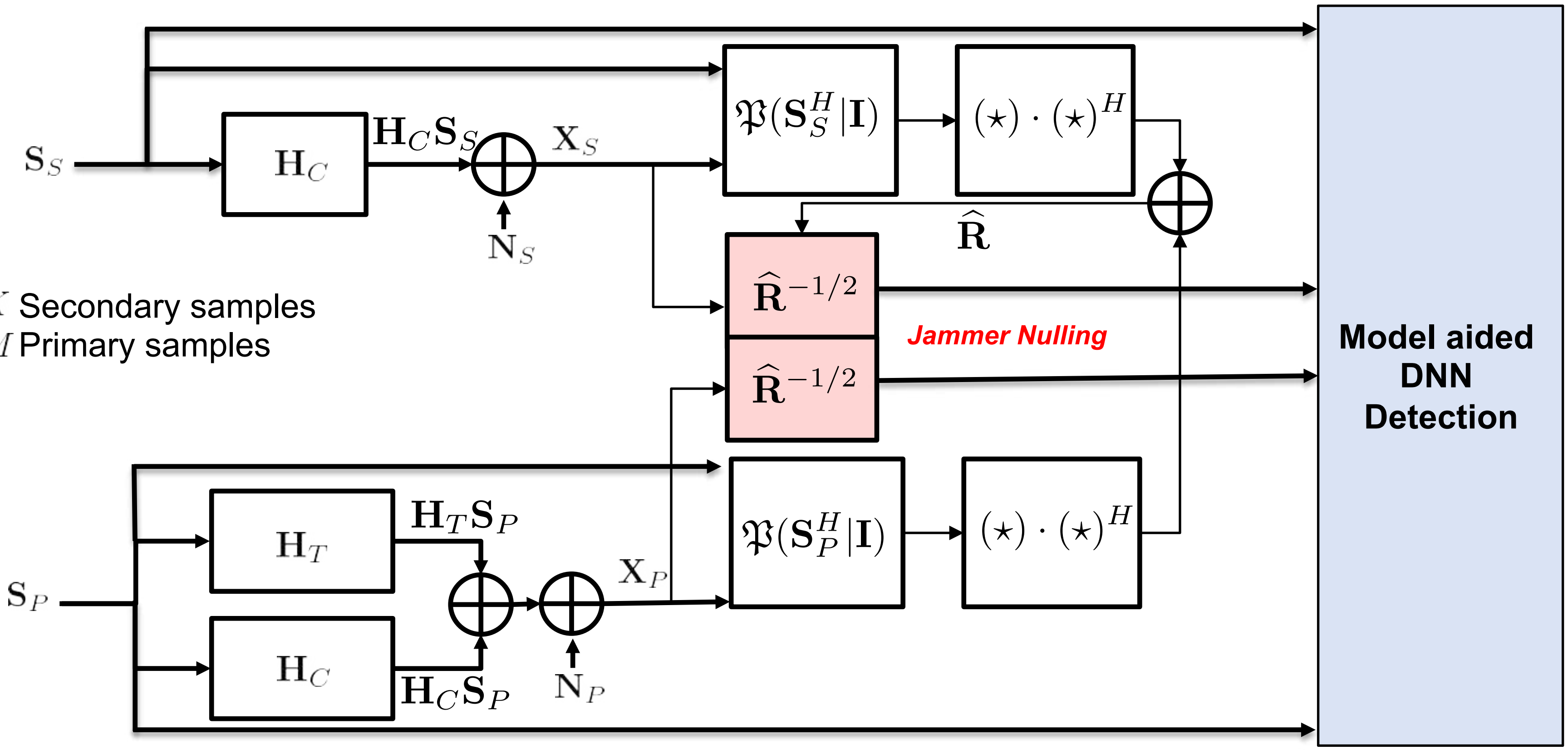
- DNNs have difficulty generalizing without extensive amounts of training data

- Model-based GLRT makes efficient use of available data and performs robustly
  - Uses knowledge of waveform
  - Performs clutter removal / nulling
  - Whitens / nulls waveform independent colored noise

- DNN architecture modified to incorporate operations similar to GLRT results in significant improvement in DNN performance and convergence rate

- DNN can ultimately outperform GLRT after full “transfer learning”

Cognitive Radar Detection: Next Step



K Secondary samples
M Primary samples

Publications

- T. Ali and C. D. Richmond, “Optimal Target Detection for Random Channel Matrix-Based Cognitive Radar/Sonar,” in 2021 IEEE Radar Conference (RadarConf21), 2021, pp. 1–6.

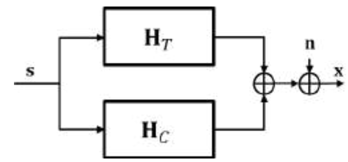
- T. Ali, A. S. Bondre, C. D. Richmond, "Adaptive Detection Algorithms for Channel Matrix-Based Cognitive Radar/Sonar," 2022 IEEE Radar Conference (RadarConf22), 2022, pp. 1-6.

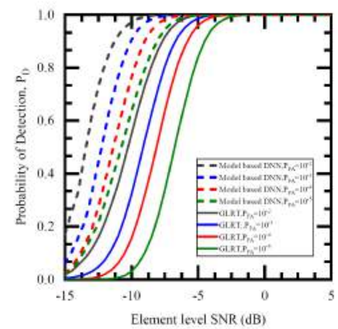
- T. Ali, A. S. Bondre, and C. D. Richmond, "Model-Aided Deep Learning-Based Target Detection for Channel Matrix-Based Cognitive Radar/Sonar," The Journal of the Acoustical Society of America, Vol. 151, No. A100, ASA Meeting, May 2022.

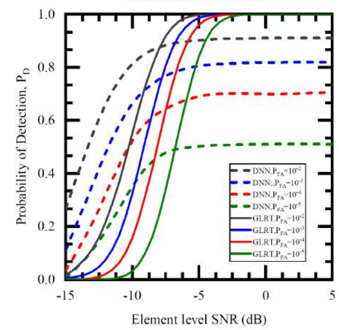
Questions & Comments Welcome

Summary

- **DNN-based target detection algorithm for channel matrix-based Cognitive Radar Framework**
- **Leveraging knowledge from GLRT derived for same framework**
- **Improvement in detection performance compared to model based GLRT algorithm and data driven DNN algorithm**
- **Future work:**
  - **Making the architecture more robust to colored noise**
  - **Integrate DRL based transmitter**







Adaptive Matched Filter (AMF) Detector

Form the optimal Neyman-Pearson test statistic, that is, the LRT.

Assume complex Gaussian statistics

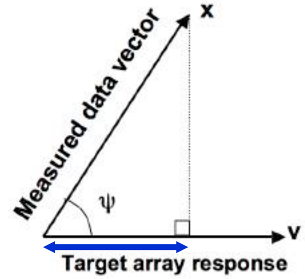
$$H_0: \quad p_{H_0} = \pi^{-N}|\mathbf{R}|^{-1}\exp\left[-\mathbf{x}^H\mathbf{R}^{-1}\mathbf{x}\right]$$

$$H_1: \quad p_{H_1} = \pi^{-N}|\mathbf{R}|^{-1}\exp\left[-(\mathbf{x}-S\mathbf{v})^H\mathbf{R}^{-1}(\mathbf{x}-S\mathbf{v})\right]$$

→ **Likelihood Ratio Test** $= \left[\frac{\max_S p_{H_1}}{p_{H_0}}\right] = t_{MF} = \frac{|\mathbf{v}^H\mathbf{R}^{-1}\mathbf{x}|^2}{\mathbf{v}^H\mathbf{R}^{-1}\mathbf{v}}$ ***Matched Filter*** *(Weiner Soln)*

Since* R *unknown use Sample Covariance:

$$\sum_{l=1}^{L}\mathbf{x}(l)\mathbf{x}^H(l) = \widehat{\mathbf{R}} \longrightarrow \mathbf{R} \quad \Rightarrow \quad t_{AMF} = \frac{|\mathbf{v}^H\widehat{\mathbf{R}}^{-1}\mathbf{x}|^2}{\mathbf{v}^H\widehat{\mathbf{R}}^{-1}\mathbf{v}}$$



Known as the *Adaptive Matched Filter (AMF)* *detector*

Return

Generalized Likelihood Ratio Test (GLRT)

Form the LRT based on the totality of data:

Test Cell

Training

$$[\mathbf{x}|\mathbf{x}(1)|\mathbf{x}(2)|\cdots|\mathbf{x}(L)] \triangleq \mathbf{X}_0$$

Assume homogeneous complex gaussian statistics

$$H_0: \quad p_{H_0} = \pi^{-N(L+1)}|\mathbf{R}|^{-(L+1)}\exp\left[-\mathrm{tr}\mathbf{R}^{-1}\mathbf{X}_0\mathbf{X}_0^H\right]$$

$$H_1: \quad p_{H_1} = \pi^{-N(L+1)}|\mathbf{R}|^{-(L+1)}\exp\left[-\mathrm{tr}\mathbf{R}^{-1}(\mathbf{X}_0-\mathbf{M})(\mathbf{X}_0-\mathbf{M})^H\right]$$

where $\mathbf{M} = [\ S\mathbf{d} \mid \mathbf{0}\ ]$

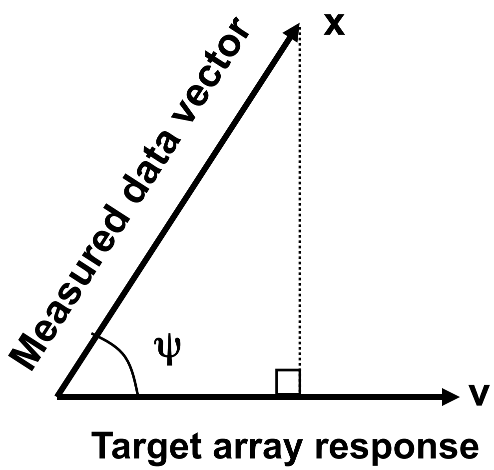
Maximize likelihood functions over all unknown parameters:

$$t_{GLRT} = \left[\frac{\max_{S,\mathbf{R}} p_{H_1}}{\max_{\mathbf{R}} p_{H_0}}\right]^{\frac{1}{L+1}} = \frac{1+\mathbf{x}^H\widehat{\mathbf{R}}^{-1}\mathbf{x}}{1+\mathbf{x}^H\widehat{\mathbf{R}}^{-1}\mathbf{x} - \dfrac{|\mathbf{v}^H\widehat{\mathbf{R}}^{-1}\mathbf{x}|^2}{\mathbf{v}^H\widehat{\mathbf{R}}^{-1}\mathbf{v}}}$$

Known as Kelly's / Khatri's *GLRT*

Return

Adaptive Coherence Estimator (ACE)

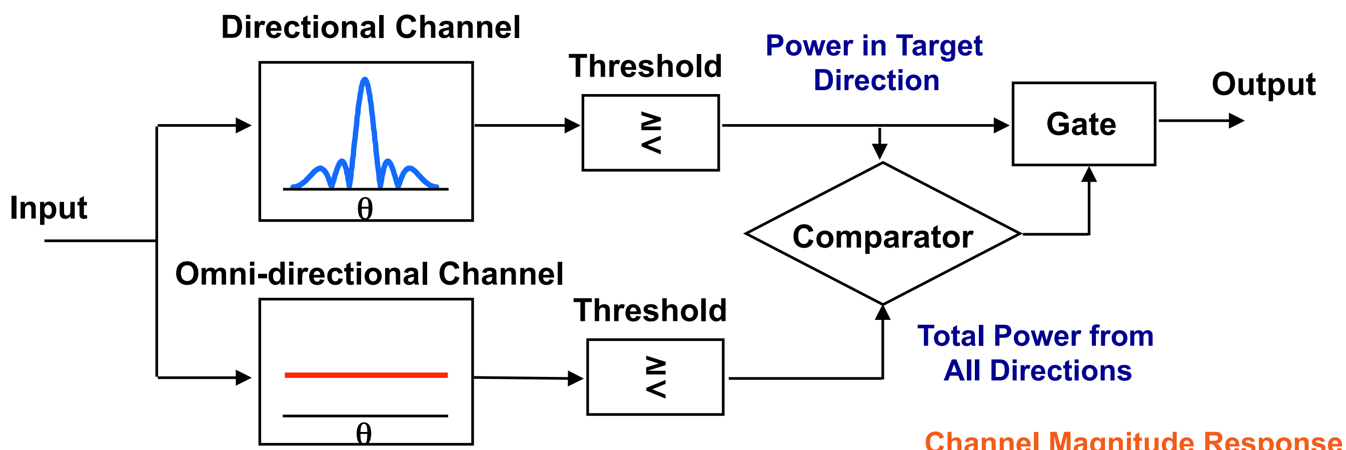


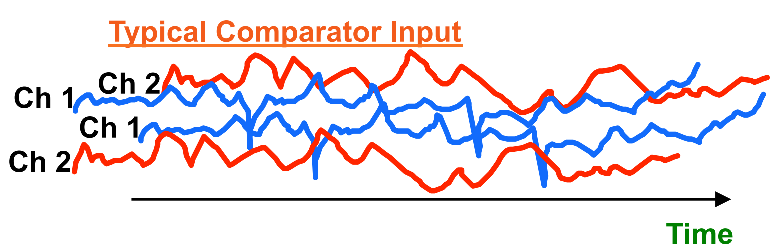
- **ACE statistic compares energy projected onto v to total power in x**
- **Inner product space defined wrt inverse of data covariance**
  - **in whitened space**

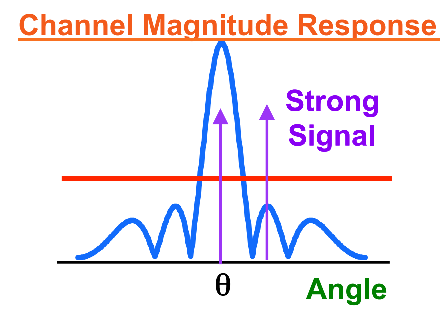
$$t_{ACE} = \frac{|\mathbf{v}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}|^2}{\mathbf{v}^H \widehat{\mathbf{R}}^{-1} \mathbf{v} \cdot \mathbf{x}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}} = |\cos\psi|^2$$

Return

Classical Sidelobe Blanking







Return

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2-D Adaptive Sidelobe Blanker (ASB) Detector

Step 1: Beamforming

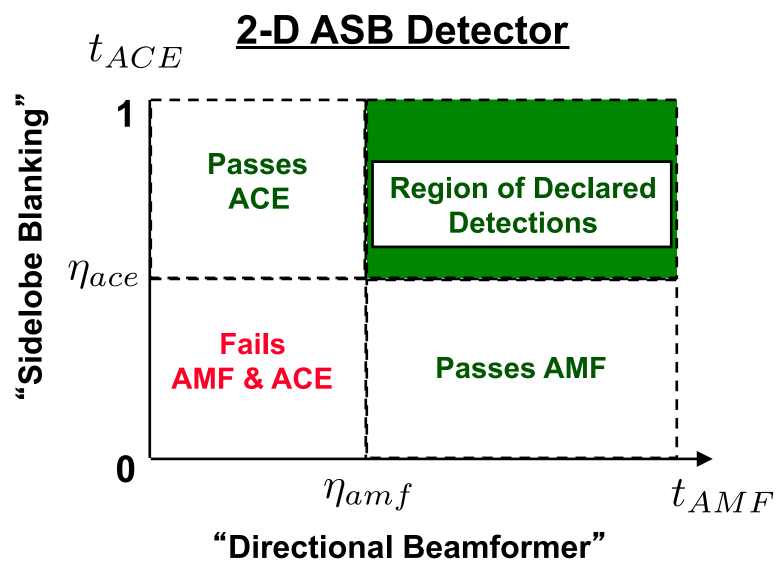
$$t_{AMF} > \eta_{amf}$$

Power in Target Direction

Step 2 : "Sidelobe Blanking"

$$t_{AMF} > \eta_{ace} \cdot \mathbf{x}^H \widehat{\mathbf{R}}^{-1} \mathbf{x}$$

Power in Target Direction | **Total Power From All Directions**



Return

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Model Free DNN based Detector

DNN Block	Layers
Branch 1 & 2	**Layer 1:** Input Layer with real and imaginary parts of the waveform and data vectors stacked **Layer 2:** Normalization Layer **Layer 3:** Dense layer with 128 perceptrons, activation: tanh **Layer 4:** Dense layer with 64 perceptrons, activation: tanh
Detection Block	**Layer 1:** Concatenation Layer **Layer 2:** Dense layer with 128 perceptrons, activation: relu **Layer 3:** Dense layer with 64 perceptrons, activation: relu **Layer 4:** Dense with 01 perceptron, activation: softmax
Threshold	Determined using Monte Carlo Simulations corresponding to particular PFA

Model Aided DNN based Detector

DNN Block	Layers
Clutter Channel Estimation Block	**Layer 1:** Input Layer with real and imaginary parts of the secondary waveform and data vectors stacked **Layer 2:** Normalization Layer **Layer 3:** Dense layer with 128 perceptrons, activation: tanh **Layer 4:** Dense layer with 64 perceptrons, activation: tanh **Layer 5:** Dense layer with $2 \times l \times l$ perceptrons, activation: linear **Layer 6:** Reshape Layer
Detection Block	**Layer 1:** Input Layer with real and imaginary parts of the primary waveform and data vectors stacked **Layer 2:** Normalization Layer **Layer 3:** Dense layer with 128 perceptrons, activation: relu **Layer 4:** Dense layer with 64 perceptrons, activation: relu **Layer 5:** Dense with 01 perceptron, activation: softmax
Threshold	Determined using Monte Carlo Simulations corresponding to particular PFA